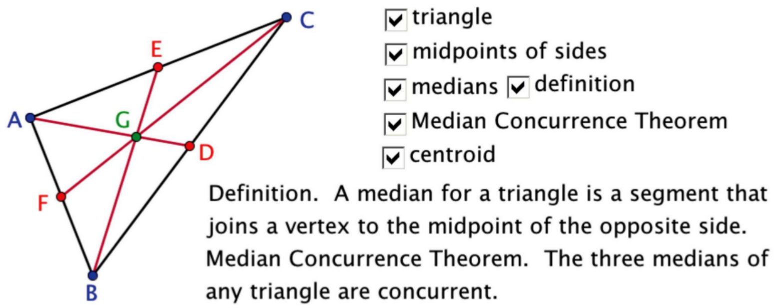
MATH1235: Python Programming & Mathematical Software

GeoGebra – GeoGebra Tools & Check Boxes, and More on Triangles

1.) GeoGebra Tools & Check Boxes:

a) Create a GeoGebra tool that accepts two points as input objects and whose output is a square together with its midpoint/centroid. Save your tool for future use.

b) Make a sketch that illustrates the Median Concurrence Theorem. Add hide/show check boxes for the triangle, the midpoints of the side, the medians, and the centroid. Add text boxes and check boxes for the definitions of a median as well as the statement of the Median Concurrence Theorem. When all boxes are checked, your sketch should look something like this:



2.) Vecten configuration and the Vecten point:

a) Construct a triangle $\triangle ABC$. Using your own tool from 1.) a), attach to each side a square towards the outside of the triangle (together with the squares' centroid midpoint).
The figure you are getting here should look familiar if you know Pythagoras' Theorem – can you confirm Pythagoras' Theorem in your sketch? The configuration of a right triangle together with the three squares is sometimes called the bride's chair, *for a general triangle we speak of the* Vecten configuration.

b) For each vertex of $\triangle ABC$, construct the line joining it to the center of the square on the opposite side. Verify that the lines are always concurrent.
The point of concurrency is called the Vecten point.

c) Experimentally check that the Vecten point is the orthocenter of the triangle formed by the midpoints of the three squares.

3.) Circumcenter and circumcircle:

a) Construct a triangle $\triangle ABC$, its circumcircle, and measure the circumradius R of $\triangle ABC$.

b) Recall the law of sines:

$$\frac{BC}{\sin(\angle BAC)} = \frac{AC}{\sin(\angle ABC)} = \frac{AB}{\sin(\angle ACB)}$$

Confirm the law of sines using your sketch.

c) Is there a relationship between the circumradius R and the fractions appearing in the law of sines?
Make notes on your observations in your online journal.

4.) Angle bisectors, the incenter, the inscribed circle and the Gergonne point:

a) Construct a triangle and the bisectors of its interior angles. Verify that the three angle bisectors are always concurrent. The point of concurrency is called the *incenter* of the triangle. Experiment with triangles of different shapes to determine whether the incenter can ever be on the triangle or outside the triangle.
Make notes on your observations in your online journal.

b) Construct another triangle and its incenter. Label the incenter I; keep it visible but hide the angle bisectors. Experiment with the triangle and the incenter to answer the following questions: Are there triangles for which the incenter equals the circumcenter? What shape are they? Are there triangles for which the incenter equals the centroid? What shape are they?
Make notes on your observations in your online journal.

c) Hide the angle bisectors. For each side of the triangle, construct a line that passes through the incenter and is perpendicular to the sideline. Mark the feet of the perpen- diculars and label them X, Y and Z such that the point on $\overline{BC}$ opposite to vertex A is labelled X, the point on $\overline{AC}$ opposite to vertex B is labelled Y, and the point on $\overline{AB}$ opposite to vertex C is labelled Z.

d) Measure the distances IX, IY, and IZ and observe that they are equal. This number is called the *inradius* of the triangle.

e) Hide the perpendiculars in your sketch from the second to last exercise, but keep the points I, X, Y, and Z visible. Construct the circle with center I and radius equal to the inradius. Observe that this circle is tangent to each of the sides of the triangle. This circle is the *inscribed circle* for the triangle.

f) Construct the segments $\overline{AX}$, $\overline{BY}$, and $\overline{CZ}$. Observe that they are concurrent, regardless of the shape of the triangle. The point of concurrency is called the Gergonne point of the triangle. It is denoted Ge. (Dont confuse it with the centroid G.)

g) For which triangles is $I = Ge$?
Make notes on your observations in your online journal.

h) Which of the points I and Ge, if any, lies on the Euler line? Do the points you have identified lie on the Euler line for every triangle, or only for some triangles?
Make notes on your observations in your online journal.