MATH1235: Python Programming & Mathematical Software

GeoGebra – Geometry: The Classical Triangle Centers

1.) Medians and the centroid:

a) Construct a triangle $\triangle ABC$. Construct the midpoints of the sides of $\triangle ABC$. Label the midpoints D, E, and F in such a way that D lies on the side opposite A, E lies on the side opposite B, and F lies on the side opposite C. Construct the medians for $\triangle ABC$. What do the medians have in common? Use the "drag test" to verify that this continues to be true when the vertices of the triangle are moved around in the plane.

b) In the preceding exercise you should have discovered that the three medians are concurrent (they have an interior point in common). Mark the point of intersection and label it G. Measure AG and GD, and then calculate AG/GD. Make an observation about the ratio. Now measure BG, GE, CG, and GF, and then calculate BG/GE and CG/GF. Leave the calculations displayed on the screen while you move the vertices of the triangle. Make an observation about the ratios.
Note your observation in your online journal.

c) Construct a triangle $\triangle ABC$, then construct the midpoints of the sides BC and AC and label them as in the first part of this question. Construct the medians AD and BE and the segment DE. Check, by measuring angles, that $\triangle ABC \sim \triangle EDC$. Then check that $\triangle ABG \sim \triangle DEG$.

d) Can you prove that $\triangle ABC \sim \triangle EDC$ and that $AB = 2 \cdot ED$? Which theorems from elementary geometry are you using for this?

e) Also prove that $\triangle ABG \sim \triangle DEG$.

f) Use the two preceding exercises to prove that $AG = 2 \cdot GD$ and $BG = 2 \cdot GE$. Explain how this allows you to conclude that $CG = 2 \cdot GF$ as well.

g) Explain why the centroid is the center of mass of the triangle. In other words, explain why a triangle made of a rigid, uniformly dense material would balance at the centroid.
Give your explanation in your online journal.

2.) Altitudes and the orthocenter:

a) Construct a triangle and construct its three altitudes. Observe that no matter how the vertices of the triangle are moved around in the plane, the altitudes continue to concur.

b) Construct another triangle. Mark the orthocenter of your triangle and label it H. Move one vertex and watch what happens to H. Add the centroid G to your sketch and again move the vertices of the triangle. Observe that the centroid is always located inside the triangle, but that the orthocenter can be outside the triangle or even on the triangle.

c) Determine by experimentation the shape of triangles for which the orthocenter is outside the triangle. Find a shape for the triangle so that the orthocenter is equal to one of its vertices. Observe what happens to the orthocenter when one vertex crosses over the sideline determined by the other two vertices.
Make notes on your observations in your online journal.

d) Determine by experimentation whether or not it is possible for the centroid and the orthocenter to be the same point. If it is possible, for which triangles does this happen?
Make notes on your observations in your online journal.

3.) Perpendicular bisectors and the circumsector:

a) Construct a triangle $\triangle ABC$ and construct the three perpendicular bisectors of its sides. These perpendicular bisectors should be concurrent and should remain concurrent when subjected to the drag test. Mark the point of concurrency and label it O.

b) Move one vertex of the triangle and observe how the circumcenter changes. Note what happens when one vertex crosses over the line determined by the other two vertices. Find triangles that show that the circumcenter may be inside, on, or outside the triangle, depending on the shape of the triangle.
Make notes on your findings in your online journal.

c) Measure the distances OA, OB, and OC and make an observation about them.
Record your observation in your online journal.

d) Can you prove that the circumcenter is equidistant from the vertices of the triangle?

e) Determine by experimentation whether or not it is possible for the circumcenter and the centroid to be the same point. If it is possible, for which triangles does this happen?
Make notes on your observations in your online journal.

4.) The Euler line:

a) Construct a triangle $\triangle ABC$ and construct the centers G, H and O. Hide any lines that were used in the construction so that only the triangle and the centers are visible. Put a line through two of them and observe that the third lies on the line. Use the "drag test" to verify that G, H, and O continue to be collinear when the shape of the triangle is changed.

b) Measure the distances HG and GO. Calculate HG/GO. Leave the calculation visible on the screen as you change the shape of your triangle. Observe what happens to HG/GO as the triangle changes.
Make notes on your observations in your online journal.