MATH1235: Python Programming & Mathematical Software

GeoGebra – Functions: Derivatives, Tangents and More

1.) Graph a family of functions, e.g., $f(x) = ax^3 + bx^2 + cx + d$ or $f(x) = a \cdot x \cdot \cos(bx + c) + d$.
Depending on your version of GeoGebra, either you first have to create sliders for a, b, c and d – so that GeoGebra knows that these are numbers to be used as parameters – or GeoGebra will automatically ask you if you want to create these sliders.

2.) Attach a point A to the graph of f (so, you can move the point A along the graph, and it doesn't leave the graph), and draw the tangent line in A to f.
Can you change the colour of the tangent to red and make it a dotted line?

3.) What is the slope of this tangent line? Can you show the slope in your drawing?

4.) Draw the graph of the derivative of f'.
You should let GeoGebra calculate the derivative here – there is a command for that. What might be its name? You might use GeoGebra's online help. Also, please change the colour of the derivative, say to red.

5.) Attach a point B to the derivative that will have the same x-coordinate as the point A (so if you move A, the point B will move with it but on the derivative).
There are several ways to achieve this. Either you can input the coordinates of the point B directly using (x-ccordinate, y-coordinate) and noting that $x(A)$ will yield the x-coordinate of A (similarly, $y(A)$ will yield the y-coordinate), and you then have to figure out its y-coordinate. Or, you use a geometric construction by looking at the intersection of a parallel line to the y-axis through A with the derivative (what is going on here?). Try both!!

6.) Can you find $\int_1^3 f(x)dx$, local maxima and local minima on the interval $[1, 3]$ etc. (or on any other interval)?
Again, there are several ways to do it. E.g., there is a menu option that will display all these properties of your function, or you can use appropriate commands to find the integral and extrema (what might they be called) in a certain interval (or globally). Again, familiarize yourself with both options.

7.) At this point, your drawing might display many things, so first make most of them invisible besides A, f and the tangent line.

8.) Draw a perpendicular line to the tangent through A. This line is called the *normal* (in A to f).

9.) Define the number

$$r = \frac{(1 + (f'(x_A))^2)^{\frac{3}{2}}}{|f''(x_A)|}$$

(here, x_A denotes the x-coordinate of A; also note that you have to calculate the second derivative first in order to calculate this r). This number r is the inverse of the so-called *curvature* κ (i.e., $\kappa = \frac{1}{r}$) of f at A.
Read the Wikipedia article on "curvature". You might not understand everything, but try to get a basic understanding.

10.) Find points on the normal that have distance r from A.
There are two of them; to get them, think "circles"!

11.) Denoting the two points we just found M and N, draw two circles, one centered at M through A, and one centered at N through A. Colour the two circles differently.

12.) By zooming in on A, which of the two circles or the tangent does *approximate* the graph of the function f close to A? If you move A around, does this change? How? Find a condition (e.g., depending on the values of f or its derivatives f', f'' at x_A) that tells you which circle does approximate your function at A best?

13.) Using the condition you found, only show that one of the two circles that approximates f best in A, i.e., your drawing should only show one of the circles, but as you move A around it may change which circle exactly is shown!

14.) Read the Wikipedia article about the "Osculating Circle". Prepare a little paragraph explaining what is going on here! (And be prepared to report your findings next Monday in class!)