MATH2130: Ordinary Differential Equations

TEST ON 8 MARCH: DIFFERENTIAL EQUATION OF THE FIRST ORDER

This test is worth 20% of the final mark.
Marks for each question are given as "[x]". There are a total of 40 marks.

1.) (a) What is meant by saying that

$$p(x, y(x)) + q(x, y(x)) \cdot y'(x) = 0 \tag{1}$$

is an exact differential equation.
Clearly state the assumptions on the functions p and q. [4]

(b) State without proof a necessary and sufficient condition for the differential equation in (1) to be exact. [2]
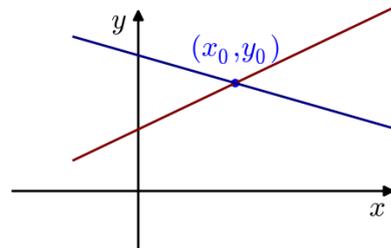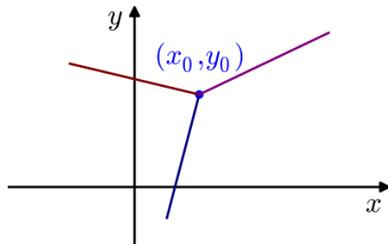
(c) Show that the differential equation

$$\frac{1}{2} y^2 + 2 y \, \mathrm{e}^x + (y + \mathrm{e}^x) \, \frac{\mathrm{d}y}{\mathrm{d}x} = 0$$

is **not** exact and find an integrating factor. Then solve the differential equation.
(**Hint**: Assume that the integrating factor is a funtion of x only.) [10]

2.) Give an example of a differential equation that has **no** solution. [2]

3.) Let $f : \mathbb{R}^2 \to \mathbb{R}$ and assume that two particular solutions of the differential equation $y' = f(x, y)$ meet in the singular point (x_0, y_0). Why are the following two cases **not** possible, i.e., why can they not merge respectively cross as depicted?

(**Hint**: Direction field.) [4]

Please turn over!

4.) We consider the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \sin(x)\, y^3.$$

(a) Solve the differential equation. [5]

(b) Find the particular solution for the initial value $y(0) = \frac{1}{3}$. On which interval is this solution defined? [3]

(c) Find the particular solution for the initial value $y(0) = \frac{1}{\sqrt{2}}$. On which interval is this solution defined? [3]

5.) For each of the following differential equations, find a substitution that turns it into a separable differential equations. State the separable differential equation, but **do not** solve it.

(a) $x'(t) = (t + x(t))^2$ [3]

(b) $t + 2\, x(t) - t\, x'(t) = 0$ [4]

Write your name and your student ID on each sheet of paper you are using.