# MATH2130: Ordinary Differential Equations

## Exercise Sheet 9: Inhomogeneous Linear Differential Equations and Cauchy Sequences

Please hand solutions in at the lecture on Monday 29th March.

1.) Solve the following inhomogeneous linear differential equations.

(a) $y'''(x) - 6\,y''(x) + 9\,y'(x) = \cos(2\,x)\ \mathrm{e}^{3\,x}$

(b) $y'''(x) - 7\,y''(x) + 19\,y'(x) - 13\,y(x) = 13\,x^3 - 57\,x^2 + 10\,x + 70$

(c) $y''(x) - 8\,y'(x) + 20\,y(x) = 5\,x\ \sin(2\,x)\ \mathrm{e}^{4\,x}$

2.) Consider the sequence of functions $(f_n)_{n\in\mathbb{N}}$ where $f_1(x) = x,\ f_2(x) = x^2,\ f_3(x) = x^3, \ldots,$ (i.e., $f_k(x) = x^k$).

(a) Show that $(f_n)_{n\in\mathbb{N}}$ is a Cauchy sequence in $(C[0, \frac{1}{2}], d_{\mathrm{sup}})$.

(b) Show that $(f_n)_{n\in\mathbb{N}}$ is **not** a Cauchy sequence in $(C[0, 1], d_{\mathrm{sup}})$.

3.) Let $(X, d)$ be a metric space. For a function $f : X \to X$, we set $f^n = f \circ f \circ \ldots \circ f$ ($n$ times).
Please show: If there exist numbers $q_n \geq 0$ such that $\lim_{n\to\infty} q_n = 0$ and $d(f^n(x), f^n(y)) \leq q_n\, d(x, y)$ for all $n$ and all $x, y \in X$, then $f$ has a unique fixed point.