MATH2130: Ordinary Differential Equations

EXERCISE SHEET 8: LINEAR DIFFERENTIAL EQUATIONS

Please hand solutions in at the lecture on Monday 22nd March.

1.) Solve the following homogeneous linear differential equations.

(a) $x'''(t) + 8\,x(t) \;=\; 0$

(b) $y'''(x) - 6\,y''(x) + 11\,y'(x) - 6\,y(x) = 0$

2.) Solve the following inhomogeneous linear differential equations.

(a) $\ddot{x}(t) + x(t) \;=\; \mathrm{e}^t$

(b) $y'''(x) + y''(x) = 6\,x + \mathrm{e}^{-x}$

3.) A second order *linear* differential equation

$$f_2(x)\,y'' + f_1(x)\,y' + f_0(x)\,y \;=\; q(x),$$

is said to be *exact* if it can be written as

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(M(x)\,\frac{\mathrm{d}y}{\mathrm{d}x} + N(x)\,y\right) \;=\; q(x),$$

i.e., if its left side can be written as the derivative of a first order *linear* expression. A necessary and sufficient condition that the equation be exact is that (you **do not** have to prove this)

$$\frac{\mathrm{d}^2 f_2}{\mathrm{d}x^2} - \frac{\mathrm{d}f_1}{\mathrm{d}x} + f_0 \equiv 0.$$

If the equation is exact, then $M(x)$ and $N(x)$ are given by

$$M(x) = f_2(x) \qquad \text{and} \qquad N(x) = f_1(x) - f_2'(x).$$

Show that the following equation is exact and solve it:

$$\left(x^2 - 2\,x\right)\,y'' + 4\,(x-1)\,y' + 2\,y \;=\; \mathrm{e}^{2\,x}\,.$$