MATH2130: Ordinary Differential Equations

Exercise Sheet 6: Linear Differential Equations of First Order

Please hand solutions in at the lecture on Wednesday 10th March.

1.) Solve the following inhomogeneous linear differential equations. Use the method 'variation of parameters'.

(i) $x'(t) + 2\,t\,x(t) = t\ \mathrm{e}^{-t^2}$.

(ii) $x'(t) + x(t)\ \tan(t) = \sin(2\,t)$

(iii) $t\,x'(t) + (t+1)\,x(t) = 3\,t^2\ \mathrm{e}^{-t}$

2.) A differential equation of the form

$$y'(x) + g(x)\,y(x) + h(x)\ (y(x))^n = 0$$

where $2 \leq n \in \mathbb{N}$, is called *Bernoulli differential equation*.

(i) Show that the substitution $z(x) = (y(x))^{1-n}$ leads to the linear differential equation

$$z'(x) + (1-n)\,g(x)\,z(x) + (1-n)\,h(x) \ = \ 0.$$

(ii) Solve the initial value problem

$$y'(x) + \frac{1}{x}\,y(x) \ = \ x\ (y(x))^2\,, \quad \text{and } y(1) = -\frac{1}{2}.$$

3.) The following differential equations can be solved by more than one method. State at least two methods each and then use one of them to solve the differential equation.

(i) $(x^2 + y^2)\,y' + 2\,x\,y = 0$

(ii) $3\ \mathrm{e}^{3\,x}\ y - 2\,x + \mathrm{e}^{3\,x}\ y' = 0$

Test on Monday 8th March at 9:10 – 10:00 in LR12 (where we usually have the lecture). Topics covered will be everything in Sections I.1 – II.4, i.e., Exercise sheets 1 – 5. You only need to bring a pen!