

MATH2130: Ordinary Differential Equations

EXERCISE SHEET 5: EXACT DIFFERENTIAL EQUATIONS AND INTEGRATING FACTORS

Please hand solutions in at the lecture on Monday 1st March.

- 1.) Solve the following differential equations. First show that $m(x, t)$ is an integrating factor.

(i) $x(t) - t x'(t) = 0$ with $m(x, t) = -\frac{t}{x^2}$.

(ii) $t x^2(t) (t x'(t) + x(t)) = 1$ with $m(x, t) = 6t$.

(iii) $x^2(t) + t^2 + t + x(t) x'(t) = 0$ with $m(x, t) = \frac{1}{t^2+x^2}$.

- 2.) Test the following differential equation for exactness and if necessary find an integrating factor; then solve the initial value problem.

$$x^2 + y^2 + x + x y y' = 0 \quad \text{and} \quad y(1) = 1.$$

- 3.) Revisit Exercise sheet 3: Try to solve those differential equations you have not done before, then read the extensive model solution and write down anything that is unclear.