MATH2130: Ordinary Differential Equations

Exercise Sheet 3: Separable and Homogeneous Differential Equations

Please hand solutions in at the lecture on Monday 15th February.

1.) We consider the differential equation $\dot{x}(t) = \sqrt{|x(t)|}$.

(i) Find solutions in the region $x > 0$.

(ii) Show: If $x(t)$ is a solution, then $y(t) = -x(-t)$ is also a solution. Use this to obtain solutions in the region $x < 0$.

(iii) Use the solutions in the previous two parts to obtain solutions for all $t \in \mathbb{R}$.

(iv) Show: One can also include a line segment $x = 0$ between the negative and the positive part of a solution $x(t)$ in (iii) and still obtain a solution defined for all $t \in \mathbb{R}$. In particular, check that such a solution is everywhere differentiable and satisfies the differential equation.

(v) Find the solutions of the initial value problem $\dot{x} = \sqrt{|x|}$ and $x(0) = 0$.

2.) Solve the initial value problem $y'(x) = \frac{1}{2}\left(y^2(x) - 1\right)$ and $y(0) = y_0$.
Find the intervals (a, b) on which the solutions $y(x)$ are defined. What happens for $y(x)$ as $x \to a^+$ or $x \to b^-$, i.e., if x approaches a from the right (b from the left)?
Hint: Treat the cases $y_0 < -1$, $-1 < y_0 < 1$ and $y_0 > 1$ separately.

3.) Solve the following homogeneous differential equations for the given initial value.

(i) $t\,x'(t) = 2\,t + x(t)$ and $x(1) = 4$. ***(this is what it should have been)***

(ii) $x'(t) = \frac{2\,x(t) - 5\,t}{x(t) - 2\,t}$ and $x(1) = 4$.

(iii) $t^2 + x^2(t) = 2\,t\,x(t)\,x'(t)$ and $x(1) = 3$.