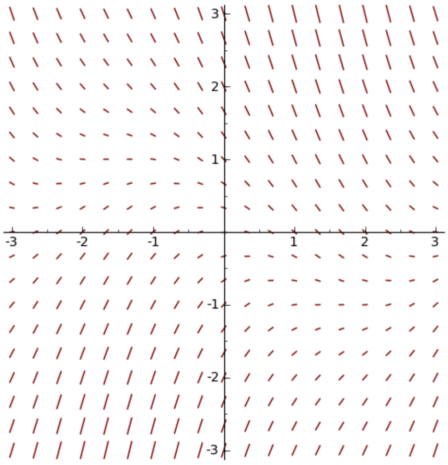
MATH2130: Ordinary Differential Equations

Exercise Sheet 2: Direction Fields and Separable Differential Equations

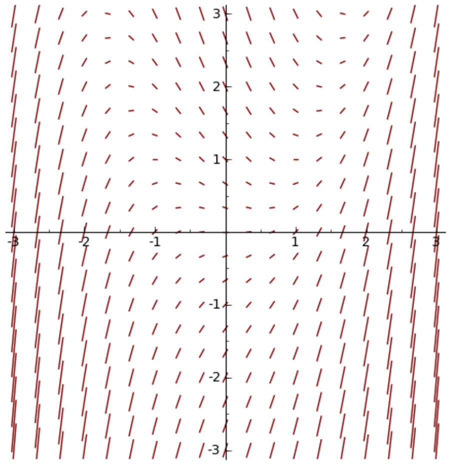
Please hand solutions in at the lecture on Monday 8th February.

1.) In this exercise, we have provided the direction field of the indicated differential equations. Sketch likely solution curves.

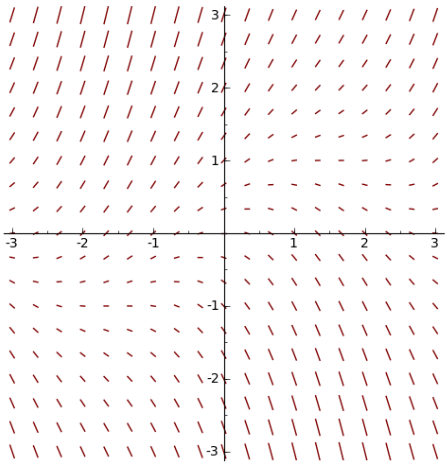
(i) $y' = -y - \sin(x)$



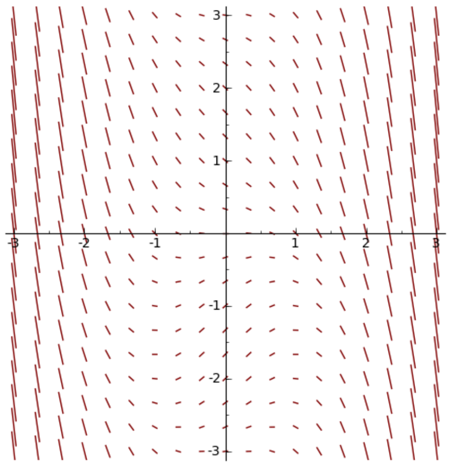
(iii) $y' = x^2 - y$



(ii) $y' = y - \sin(x)$



(iv) $y' = -x^2 - \sin(y)$



Please turn over!

2.) The differential equation $\dot{x} = r \cdot (1 - x)\, x$ is called logistic equation.

(i) Let $r = 1$. Construct the direction field for $x \geq 0$ and $t \geq 0$ and sketch some likely solutions.
Hint: Solutions where $0 < x < 1$ are different from the ones where $x > 1$. Find the isoclines.

(ii) Solve the logistic equation.
Hint: $\frac{1}{(1-x)\,x} = \frac{1}{1-x} + \frac{1}{x}$

(iii) The logistic equation is a simple mathematical model for a population growth that is limited by the available resources (time t, population x, growth rate r). Interpret the solution accordingly.

3.) Solve the following initial value problems.

(i) $t \cdot \dot{x} = 2\,x$ and $x(1) = x_0$.

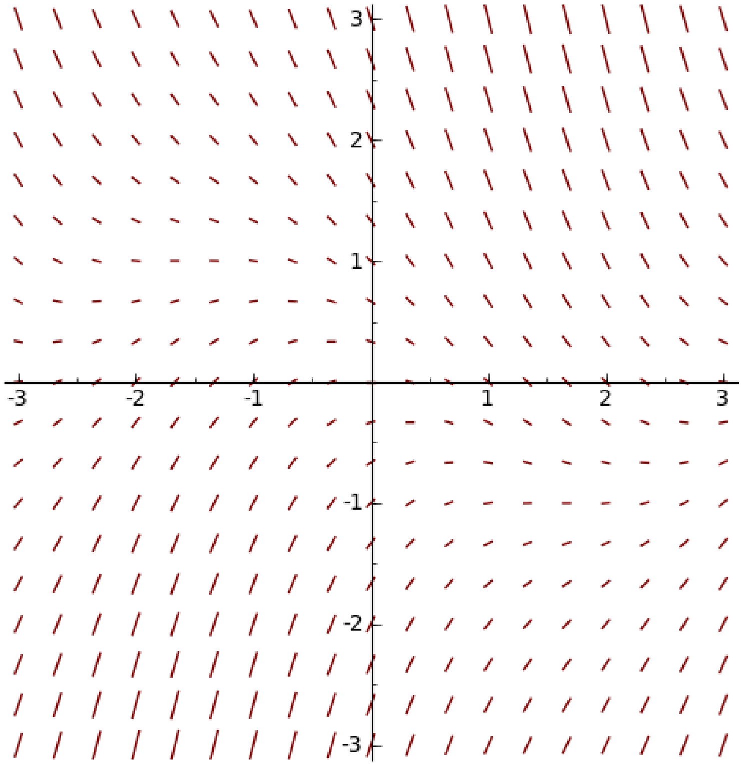
(ii) $x^2 + y^2 \cdot y' = 1$ and $y(0) = -1$.

(iii) $\dot{x} = \mathrm{e}^{t+x+3}$ and $x(0) = -3$.

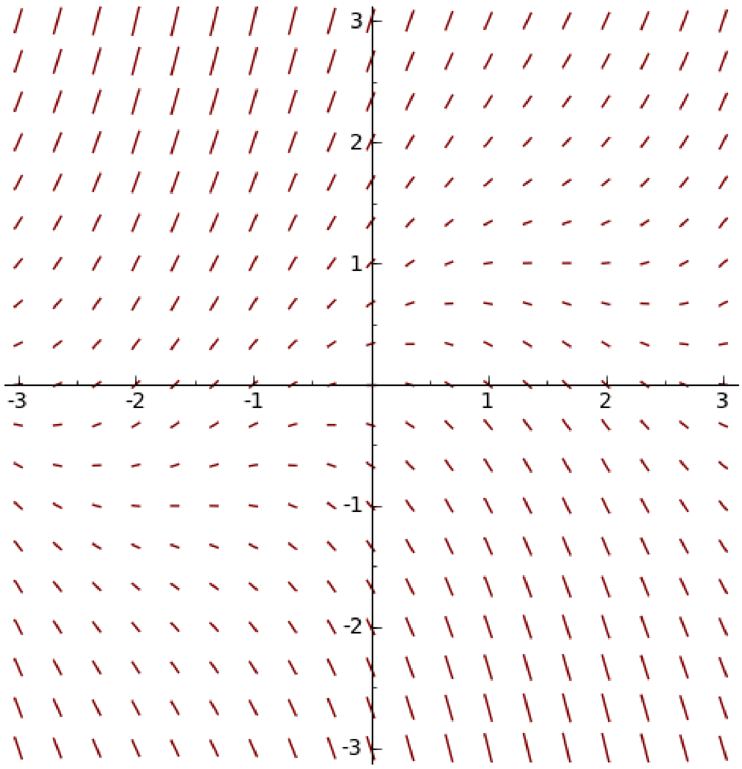
(iv) $y'(x) = \mathrm{e}^{y(x)} \cdot \sin(x)$ and $y(0) = 0$.

(v) $x'(t) + 2\,t\,x(t) = 0$ and $x(0) = x_0$.

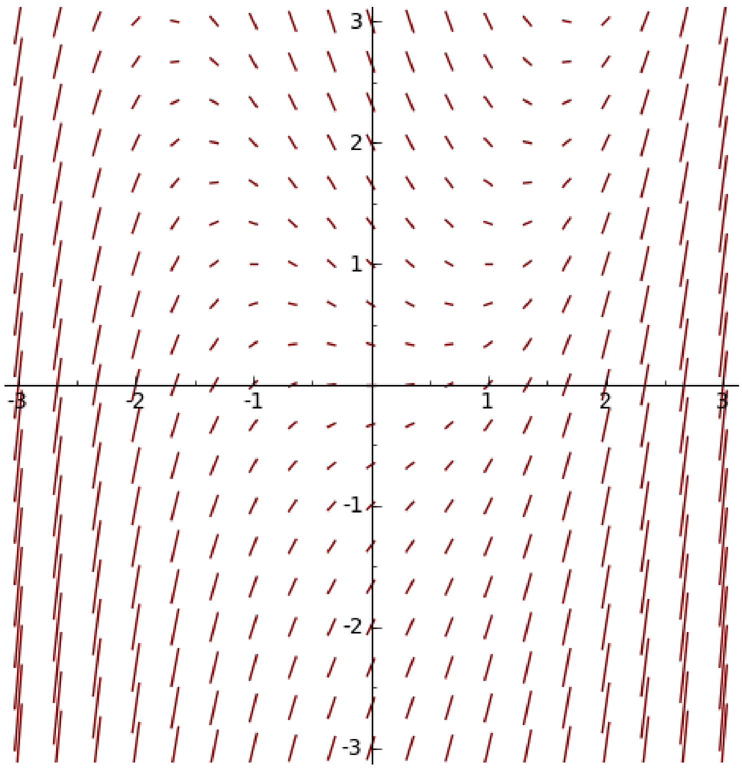
(i) $y' = -y - \sin(x)$



(ii) $y' = y - \sin(x)$



(iii) $y' = x^2 - y$



(iv) $y' = -x^2 - \sin(y)$