MATH2130: Ordinary Differential Equations

Self-Assessment Sheet 3: Separable and Homogeneous Differential Equations

1.) In class we considered the separable differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} \;=\; x \cdot (1 + y^2)$$

for which we found the general solution $y(x) = \tan\,(x^2 + c)$.

Now, consider the initial value condition $y(0) = 1$. If we solve for c, we get

$$1 \;=\; y(0) \;=\; \tan\left(0^2 + c\right) \;=\; \tan(c),$$

but – by the periodicity of tan – there are now many values $c \in \mathbb{R}$ that satisfy this equation, namely, $c = \frac{\pi}{4}, \frac{\pi}{4} \pm \pi, \frac{\pi}{4} \pm 2\,\pi, \ldots$. Which value for c should we choose?
For the solution, click on the following space:

2.) We know that the initial value problem $\frac{\mathrm{d}y}{\mathrm{d}x} \;=\; y$ and $y(0) = a$ has the unique solution $y(x) = a\,\mathrm{e}^x$. But what happens if we square this differential equation, i.e., can you solve the initial value problem $\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2 \;=\; y^2$ and $y(0) = a$?
For the solution, click on the following space:

Please turn over!

3.) Are the following differential equations homogeneous?
Click on "Evaluate" after you have ticked those that are homogeneous.

- ◯ $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{y+x}{x}$
- ◯ $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2\,y^4+x^4}{x\,y^3}$
- ◯ $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\frac{y}{x}+1}{x}$
- ◯ $\frac{\mathrm{d}y}{\mathrm{d}x} = \sin(x\,y)$
- ◯ $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{y^2+x\,y\,\sin(x/y)+x^2}{x^2}$

Evaluate