

MATH2130: Ordinary Differential Equations

SELF-ASSESSMENT SHEET 1: BASIC CONCEPTS

1.) Determine the order of the following differential equations.

Click on "Evaluate" after you have filled in the appropriate numbers.

(i) $y'' + x \cdot y'' + 2y (y')^3 + x \cdot y = 0$ has order _____.

(ii) $e^{y'''} + x \cdot y'' + y = 0$ has order _____.

Evaluate

2.) First verify that $y(x)$ satisfies the given differential equation. Then determine a value of the constant C so that $y(x)$ satisfies the given initial condition $y(x_0) = y_0$.

(i) Differential equation: $y' + y = 0$; (general) solution: $y(x) = C \cdot e^{-x}$; initial condition $y(0) = 2$.

For the solution, click on the following space:

(ii) Differential equation: $y' = x - y$; (general) solution: $y(x) = C \cdot e^{-x} + x - 1$; initial condition $y(0) = 10$.

For the solution, click on the following space:

Please turn over!

3.) Read the following formal definition of *implicit solution* and the example following it, then answer the question.

A relation $g(x, y) = 0$ is called an *implicit solution* of the differential equation $F(x, y, y', \dots, y^{(n)}) = 0$ if

- y defines an implicit function on x , i.e., there exists a function $h = h(x)$ such that $g(x, h(x)) = 0$, and
- this function $h(x)$ solves the differential equation.

In the lecture, we have discussed that the equation $g(x, y) = x^2 + y^2 - 25 = 0$ is an implicit solution for $y' \cdot y + x = 0$: $g(x, y) = 0$ defines two functions $h_1(x) = \sqrt{25 - x^2}$ and $h_2(x) = -\sqrt{25 - x^2}$ both satisfying the differential equation.

Now consider the same differential equation $y' \cdot y + x = 0$ again, and let $k(x, y) = (y - 2x) \cdot (x^2 + y^2 - 25)$. Is the relation $k(x, y) = 0$ also an implicit solution of this differential equation?

For the solution, click on the following space:

4.) Is $y(x) = \sqrt{-(1 + x^2)}$ a solution of the differential equation $y' \cdot y + x = 0$?

For the solution, click on the following space:
