

## MATH1130: Calculus II

### FIRST TEST: VECTORS IN EUCLIDEAN SPACE

This test is worth 10% of the final mark.

Each of the following questions carries equal marks.

- 1.) Let  $\mathbf{x}$  and  $\mathbf{y}$  be vectors in  $\mathbb{R}^n$ .
  - (a) State the definition of the scalar product between  $\mathbf{x}$  and  $\mathbf{y}$ .
  - (b) State the definition of the norm of  $\mathbf{x}$ .
  - (c) What does it mean to say that two vectors  $\mathbf{x}$  and  $\mathbf{y}$  are perpendicular.
  
- 2.) Let  $\mathbf{x} = (1, 1, 1, 1)$  and  $\mathbf{y} = (1, -3, -1, 5)$ .  
Find the projection  $\mathbf{w}$  of  $\mathbf{x}$  onto  $\mathbf{y}$ , and the projection  $\mathbf{u}$  of  $\mathbf{y}$  onto  $\mathbf{x}$ .  
Also verify that  $\mathbf{y}$  is perpendicular to  $(\mathbf{x} - \mathbf{w})$ .
  
- 3.) For the pair of vectors  $\mathbf{x} = (-1, 4, 1)$  and  $\mathbf{y} = (3, 2, -1)$ , find  $\mathbf{x} \times \mathbf{y}$ , and verify that  $\mathbf{x}$  is perpendicular to  $\mathbf{x} \times \mathbf{y}$  and  $\mathbf{y}$  is perpendicular to  $\mathbf{x} \times \mathbf{y}$ .  
Also, calculate  $\|\mathbf{x} \times \mathbf{y}\|$  and give a geometric interpretation of this value.
  
- 4.) Let  $\mathbf{p} = (1, -2, 2)$ ,  $\mathbf{v} = (1, 2, 2)$ ,  $\mathbf{q} = (1, 3, -1)$  and  $\mathbf{n} = (2, 1, -1)$ .  
Find the point of intersection of the line through  $\mathbf{p}$  in the direction of  $\mathbf{v}$ , and the plane through  $\mathbf{q}$  normal to  $\mathbf{n}$ .  
Explicitly check that the point of intersection is contained in the plane.
  
- 5.) For any vectors  $\mathbf{x}, \mathbf{y} \in \mathbb{R}^n$ , prove that

$$\|\mathbf{x} + \mathbf{y}\|^2 - \|\mathbf{x} - \mathbf{y}\|^2 = 4 \langle \mathbf{x}, \mathbf{y} \rangle.$$

Explain what happens if  $\mathbf{x}$  and  $\mathbf{y}$  are perpendicular.

*Write your name and your student ID on each sheet of paper you are using.*