MATH1130: Calculus II

Exercise Sheet 9: Tangent planes, extrema & integrals

Please hand solutions in at the lecture on Tuesday 30th March.

1.) Suppose f is a function such that

$$\nabla f(1,1,1) = (5,2,1).$$

Let $\boldsymbol{r}(t) = (t^2, t^{-3}, t)$. Find

$$\frac{\mathrm{d}}{\mathrm{d}t} f(\boldsymbol{r}(t)) \qquad \text{at } t = 1.$$

2.) Let $f(x,y,z) = z - \mathrm{e}^x \sin(y)$ and $\boldsymbol{p} = (\ln(3), \frac{3}{2}\pi, -3)$. Find:

(a) $\nabla f(\boldsymbol{p})$,

(b) the tangent plane to the level surface for f which passes through $\boldsymbol{p}$ at $\boldsymbol{p}$.

3.) (a) Calculate the vector that is perpendicular to the level curve l of f at $(x_0, y_0) \in l$ for

(i) $f(x,y) = x^2y^2 - 2\,y^3$ and $(x_0, y_0) = (2,1)$,

(ii) $f(x,y) = \sin(x\,y)$ and $(x_0, y_0) = (1, \frac{\pi}{2})$,

(b) Find the equation of the tangent plane to each of the following surfaces at the specific point.

(a) $x^2 + y^2 + z^2 = 49$ at $(6,2,3)$

(b) $xy + yz + zx - 1 = 0$ at $(1,1,0)$

Please turn over!

4.) Let $f(x,y) = (y - 4\,x^2)\,(y - x^2)$.

(a) Verify that $(0,0)$ is a critical point of f.

(b) Show that $\Delta(0,0) = 0$.

(c) Show that along any line through the origin, f has a local minimum at $(0,0)$.

(d) Show that along the line $\boldsymbol{\delta}(t) = (t, 3\,t^2)$, f has a local maximum at $(0,0)$. Note that this shows that $(0,0)$ is a saddle point.

5.) Evaluate each of the following iterated integrals.

(a) $\int_1^2 \int_0^2 3\,x\,y^2 \;\mathrm{d}y\,\mathrm{d}x$

(b) $\int_{-2}^2 \int_{-1}^1 (4 - \,x^2\,y^2)\;\mathrm{d}y\,\mathrm{d}x$

(c) $\int_0^2 \int_0^1 \mathrm{e}^{x+y}\;\mathrm{d}y\,\mathrm{d}x$

Second test on Monday 29th February at 12:10 – 13:00 in LT3 (where we usually have the lecture). Topics covered will be everything in Sections II.1 – III.4, i.e., Exercise sheets 5 – 8. There will be THREE (3) main questions. Full marks will be given for correct answers to TWO (2) questions. Only the best two answers will contribute towards the assessment.You only need to bring a pen!