

## MATH1130: Calculus II

### EXERCISE SHEET 5: LIMITS, DERIVATIVES AND INTEGRALS OF VECTOR FUNCTIONS

Please hand solutions in at the lecture on Tuesday 2nd March.

1.) Let  $\mathbf{r} : \mathbb{R} \rightarrow \mathbb{R}^3$  be defined by

$$\mathbf{r}(t) = \left( \frac{\sin(t)}{t}, \cos(t), 3t^2 \right).$$

Evaluate the following limits.

(a)  $\lim_{t \rightarrow \pi} \mathbf{r}(t)$

(b)  $\lim_{t \rightarrow 1} \mathbf{r}(t)$

(c)  $\lim_{t \rightarrow 0} \mathbf{r}(t)$

2.) Discuss the continuity of each of the following functions

(a)  $\mathbf{r}(t) = (t^2 + 1, \cos(2t), \sin(3t))$

(b)  $\mathbf{r}(t) = (\sqrt{t+1}, \tan(t))$

(c)  $\mathbf{r}(t) = \left( \frac{1}{t^2-1}, \sqrt{1-t^2}, \frac{1}{t} \right)$

(d)  $\mathbf{r}(t) = (\cos(4t), 1 - \sqrt{3t+1}, \sin(5t), \sec(t))$

3.) Find the derivative of each of the following functions.

(a)  $\mathbf{r}(t) = (t^3, t, 2t + 4)$

(b)  $\mathbf{r}(t) = (3t \cos(2t), 4t \sin(2t))$

(c)  $\mathbf{r}(t) = (4t^3 - 3, \sin(t), e^{-2t})$

(d)  $\mathbf{r}(t) = (e^{-t} \sin(3t), e^{-t} \cos(3t), t e^{-t})$

For (b) compare  $\frac{d}{dt} \|\mathbf{r}(t)\|$  and  $\|\mathbf{r}'(t)\|$ .

For (d) let  $f(t) = e^{-t}$  and  $\mathbf{p}(t) = (\sin(3t), \cos(3t), t)$  and verify that  $\frac{d}{dt} (f(t) \cdot \mathbf{p}(t)) = \frac{df}{dt}(t) \cdot \mathbf{p}(t) + f(t) \cdot \frac{d\mathbf{p}}{dt}(t)$ .

*Please turn over!*

4.) Evaluate the following integrals.

(a)  $\int_{-1}^1 \mathbf{r}(t) dt$  where  $\mathbf{r}(t) = (t, -t e^{-t})$ .

(b)  $\int_1^2 \mathbf{r}(t) dt$  where  $\mathbf{r}(t) = \left(\frac{1}{\sqrt{t^3}}, -\sqrt{t}\right)$ .

(c)  $\int_1^2 \mathbf{r}(t) dt$  where  $\mathbf{r}(t) = \left(\frac{1}{t}, -t\sqrt{t^2+1}, (t-1)\sin(t/2)\right)$ .