

MATH1130: Calculus II

EXERCISE SHEET 4: LINEAR FUNCTIONS AND MATRICES

Please hand solutions in at the lecture on Tuesday 23rd February. Attempting Exercise 1 is worth 1% of the final mark.

1*.) Suppose $\mathbf{T} : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ and $\mathbf{S} : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ are defined by

$$\mathbf{T}(x, y, z) = (2x + 3y, y - x + 2z, x + 2y - z)$$

and

$$\mathbf{S}(x, y, z) = (2x + 4y - 3z, z + y + x, 3x - y + 4z).$$

Find matrices for the following linear functions.

$$\begin{array}{lll} \text{(i)} \ 3\mathbf{T} & \text{(ii)} \ \mathbf{T} + \mathbf{S} & \text{(iii)} \ 2\mathbf{T} - \mathbf{S} \\ \text{(iv)} \ \mathbf{S} + 2\mathbf{T} & \text{(v)} \ \mathbf{S} \circ \mathbf{T} & \text{(vi)} \ \mathbf{T} \circ \mathbf{S} \end{array}$$

2.) Suppose $\mathbf{T} : \mathbb{R}^n \rightarrow \mathbb{R}^m$ is a linear function for some $n, m \in \mathbb{N}$.

(i) If $\mathbf{T}(1, 1) = (3, \pi, 0)$ and $\mathbf{T}(0, 1) = (4, 0, 1)$, find the matrix representative of \mathbf{T} .

(ii) If $\mathbf{T}(1, 1, 0) = (3, 3)$, $\mathbf{T}(0, -1, 1) = (1, 0)$ and $\mathbf{T}(1, 1, -1) = (1, 2)$, find the matrix representative of \mathbf{T} .

(iii) If $\mathbf{T}(0, 1, 1, 0) = (3, 5)$, $\mathbf{T}(0, 1, -1, 0) = (5, 3)$ and $\mathbf{T}(0, 0, 0, -1) = (3, 3)$, find *all possible* matrix representative of \mathbf{T} .

Hint: Use that \mathbf{T} is linear; how does one find the columns of the matrix representative?

3.) Let

$$R_\theta = \begin{pmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{pmatrix}.$$

(i) Show that for any $\mathbf{x} \in \mathbb{R}^2$ we have $\|R_\theta \mathbf{x}\| = \|\mathbf{x}\|$.

(ii) For any $\mathbf{x} \in \mathbb{R}^2$, let α be the angle between \mathbf{x} and $R_\theta \mathbf{x}$. Show that $\cos(\alpha) = \cos(\theta)$.

Convince yourself that $R_\theta \mathbf{x}$ is the rotation of \mathbf{x} in the origin through an angle θ .

First test on Monday 22nd February at 12:10 – 13:00 in LT3 (where we usually have the lecture). Topics covered will be everything in Sections I.1 – I.4, i.e., Exercise sheets 1 – 3. You only need to bring a pen!