

MATH1130: Calculus II

EXERCISE SHEET 3: LINES AND PLANES

Please hand solutions in at the lecture on Tuesday 16th February.

- 1.) Find a parametric representation for the line passing through the following pairs of points.

(i) $\mathbf{x} = (1, 3, 1)$ and $\mathbf{y} = (-4, 1, 2)$

(ii) $\mathbf{x} = (-1, 5, 3)$ and $\mathbf{y} = (-2, 4, 7)$

Check if these two lines intersect.

- 2.) Let $\mathbf{x} = (1, 3, -1)$ and $\mathbf{y} = (-4, 5, 2)$. Determine the coordinates of the following points:

(i) The midpoint of the line segment between \mathbf{x} and \mathbf{y} .

(ii) The point lying one-fifth of the way from \mathbf{x} to \mathbf{y} .

- 3.) Find the normal vector and the (normal) equation for the plane

(i) passing through the three points $(2, 1, 1)$, $(3, -1, 1)$ and $(4, 1, -1)$.

(ii) with parametric equation

$$\mathbf{P}(t, s) = t(1, 2, 1) + s(3, 1, -1) + (1, -1, 1)$$

where $t, s \in \mathbb{R}$.

- 4.) Find the cosine of the angle between the following planes:

(i) $x + y + z = 1$ and $x - y - z = 5$.

(ii) $2x + y + z = 3$ and $-x - y + z = \pi$.

- 5.) Let $\mathbf{p} = (1, 3, -1)$, $\mathbf{q} = (1, -1, 2)$, $\mathbf{n} = (1, 2, 2)$ and $\mathbf{v} = (2, 1, -1)$. Find the point of the intersection of the line through \mathbf{p} in the direction of \mathbf{v} , and the plane through \mathbf{q} perpendicular to \mathbf{n} .

- 6.) Show that if $\mathbf{x} \perp \mathbf{y}$ and $\mathbf{x} \perp \mathbf{z}$ (i.e., \mathbf{x} is perpendicular to both \mathbf{y} and \mathbf{z}), then $\mathbf{x} \perp (a\mathbf{y} + b\mathbf{z})$ for any scalars $a, b \in \mathbb{R}$.