MATH1130: Calculus II

Exercise Sheet 11: Change of Variables and Vector fields

Please hand solutions in at the lecture on Tuesday 13th April.

1.) Use polar coordinates to evaluate the following integrals.

(a) Let $D \subset \mathbb{R}^2$ be the annular region which lies between the circles with equation $x^2 + y^2 = 1$ and $x^2 + y^2 = 16$, and evaluate

$$\iint_D \frac{1}{x^2 + y^2} \, \mathrm{d}x \, \mathrm{d}y.$$

(b) With the same region D as in (a), evaluate

$$\iint_D \frac{x + y^2}{x^2 + y^2} \, \mathrm{d}x \, \mathrm{d}y.$$

(c) Similar to (a), but let $D \subset \mathbb{R}^2$ be the annular region in the first quadrant which lies between the circles with equation $x^2 + y^2 = 1$ and $x^2 + y^2 = 16$ (i.e., $D = \{(x, y) \in \mathbb{R}^2 \,|\, x \geq 0, y \geq 0, x^2 + y^2 \geq 1, x^2 + y^2 \leq 16\}$, and evaluate

$$\iint_D \frac{1}{x^2 + y^2} \, \mathrm{d}x \, \mathrm{d}y.$$

(d) Evaluate

$$\iint_D x \, \sin(\sqrt{x^2 + y^2}) \, \mathrm{d}x \, \mathrm{d}y$$

where D is the region described by $x \geq 0$, $y \geq 0$ and $x^2 + y^2 \leq 1$.

2.) Let S be the region bounded by $y = -x$, $y = \frac{1}{3}$, $y = 2\,x$ and $y = 2\,x - 1$. Evaluate

$$\iint_S \frac{x + y}{(2\,x - y + 1)} \, \mathrm{d}x \, \mathrm{d}y$$

3.) Find the set of points in $\mathbb{R}^2$ for which the vector field

$$\boldsymbol{f}(x, y) = \left(2\,x \sin(x - y), \frac{4\,x + 3\,y}{2\,x - y}\right).$$

is continuous.

Please turn over!

4.) Find the derivative for each of the following functions at the specified point $\boldsymbol{c}$.

(a) $\boldsymbol{f}(x, y) = (x^2 + y^2, 3\,x\,y)$ at $\boldsymbol{c} = (1, 2)$

(b) $\boldsymbol{f}(x, y, z) = (\sin(x + y + z), x\,y\,\cos(z))$ at $\boldsymbol{c} = (0, \frac{\pi}{4}, \frac{\pi}{4})$

(c) $\boldsymbol{f}(x, y) = (3\,x^2 + y, x - y, 4\,x\,y^2, 4\,y - x)$ at $\boldsymbol{c} = (-1, 3)$