

MATH1130: Calculus II

EXERCISE SHEET 10: INTEGRATION

Please hand solutions in at the lecture on Tuesday 6th April. Attempting Exercise 1 is worth 1% of the final mark.

- 1.) (a) Sketch the domain of integration and evaluate the iterated integral

$$\int_0^1 \int_y^1 x^2 e^{x \cdot y} dx dy.$$

- (b) Evaluate the iterated integral

$$\int_0^1 \int_y^1 \frac{\sin(x)}{x} dx dy.$$

- 2.) Interpret each of the following iterated integrals as a double integral $\int_D f dA$ for the appropriate region D , sketch D , and change the order of integration. (You may assume that f is continuous.)

(a) $\int_0^1 \int_0^{2x} f(x, y) dy dx$

(b) $\int_1^2 \int_{y^2}^4 f(x, y) dx dy$

(c) $\int_0^1 \int_{x^2}^x f(x, y) dy dx$

(d) $\int_{-1}^1 \int_0^{\sqrt{1-y^2}} f(x, y) dx dy$

- 3.) For each of the following, evaluate the iterated integrals and sketch the region of integration.

(a) $\int_0^2 \int_0^y (x y^2 - x^2) dx dy$

(b) $\int_0^1 \int_{x^2}^{x^2+y^2} (x^2 + y^2) dy dx$

(c) $\int_0^1 \int_0^{y^2} x y e^{-x-y} dx dy$

Please turn over!

4.) Evaluate each of the following iterated integrals.

(a) $\int_1^2 \int_0^3 \int_{-2}^2 (4 - x^2 - z^2) \, dy \, dx \, dz$

(b) $\int_{-2}^3 \int_{-1}^2 \int_0^2 3 \cdot x y z \, dx \, dy \, dz$

(c) $\int_0^4 \int_0^x \int_0^{x+y} (x^2 - y z) \, dz \, dy \, dx$