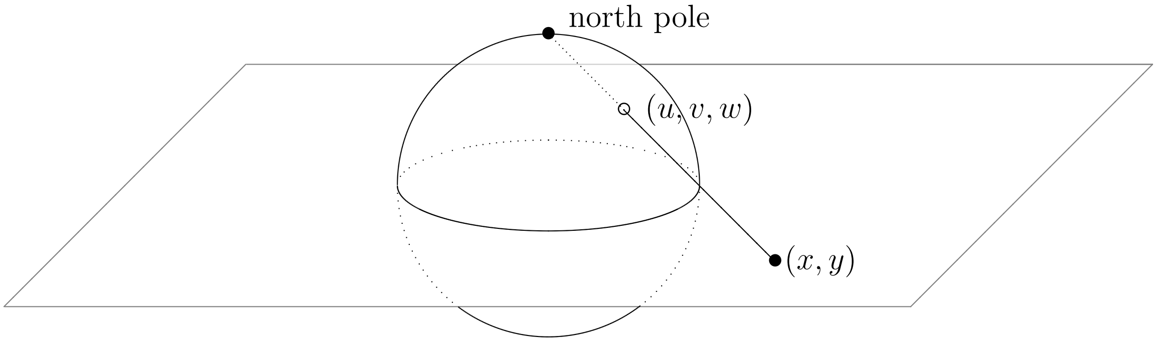
MA30056: Complex Analysis

I.4 Stereographic Projection (Not Examinable!)

A complex number $z = x + iy \in \mathbb{C}$ can be represented as point (x, y) in the plane $\mathbb{R}^2$. One can also associate a point (u, v, w) on the unit sphere $\mathbb{S} = \{(u, v, w) \in \mathbb{R}^3 \mid u^2 + v^2 + w^2 = 1\}$, called the ________________________, with a given point (x, y) in the plane. The associated mapping is called *stereographic projection*.



- A point (u, v, w) on the sphere corresponds to (x, y) if the north pole $(0, 0, 1)$, (u, v, w) and $(x, y, 0)$ are on a line.
- The equator of the sphere corresponds to the ________________________ in the plane.
- The south pole $(0, 0, -1)$ corresponds to ________________________.

Definition. The *stereographic projection*, which projects a point $(u, v, w) \in \mathbb{S} \setminus \{0, 0, 1\}$ to a point of the (complex) plane $z = x + iy \in \mathbb{C} \cong \mathbb{R}^2$, and its inverse are given by the following maps:

$$u = \frac{2\,x}{|z|^2 + 1}, \qquad v = \frac{2\,y}{|z|^2 + 1}, \qquad w = \frac{|z|^2 - 1}{|z|^2 + 1}$$

and

$$x = \frac{u}{1 - w}, \qquad y = \frac{v}{1 - w}.$$

$\mathbb{C}$ $\overset{\text{continuous}}{\longleftrightarrow}$	$\mathbb{S} \setminus \{(0, 0, 1)\}$
with the topology of $\mathbb{R}^2$	with the topology of $\mathbb{R}^3$, i.e., metric is given by the Euclidean metric in $\mathbb{R}^3$ (*chordal metric* on $\mathbb{S}$)
is complete	
closed and open	
not compact	

Definition. $\widehat{\mathbb{C}} = \mathbb{C} \cup \{\infty\}$ is called the *extended complex plane*, where ∞ denotes the *point at infinity* (its image point on the Riemann sphere is the north pole).