MA30056: Complex Analysis

SELF-ASSESSMENT SHEET 9: LAURENT SERIES & SINGULARITIES

1.) a) How many different Laurent series about the point 1 does the function $z \mapsto \frac{1}{z(z-1)(z-2)}$ have? Where are these series convergent?
For the solution, click on the following space:

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b) Answer the same question for the same function, but for the point 2.
For the solution, click on the following space:

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2.) Write down *all* the points at which the following functions have singularities and guess their type (removable/pole/essential singularity).
Click on the correspond line for the solution!

a) $f(z) = z \sin \frac{1}{z}$ ______________________________

b) $f(z) = \frac{z+1}{(z-1)\,(z^2+2iz-1)}$ ______________________________

c) $f(z) = \frac{z-i}{z^2+1}$: ______________________________

d) $f(z) = \begin{cases} z & \text{if } z \neq 0, \\ 1 & \text{if } z = 0. \end{cases}$ ______________________________

3.) Suppose that the function f has a singularity at 0. For each of the following cases, state what kind of singularity it is if the given statement is true.
Click on the correspond line for the solution!

a) $f(z) = \frac{g(z)}{z}$, $z \neq 0$, where g is entire and $g(z) = 0$.

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b) $f(z) = \frac{g(z)}{z}$, $z \neq 0$, where g is entire and $g(z) \neq 0$.

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c) $\lim_{z \to 0} z^m f(z)$ does not exist for any positive integer m.

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d) $\lim_{z \to 0} z^m f(z) = 0$, where m is an integer with $m \geq 2$.

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e) $f(z) = \frac{1}{g(z)}$, $z \neq 0$, where g is entire and has just one zero, a zero of order 1 at 0.

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