MA30056: Complex Analysis

Self-Assessment Sheet 8: Power series

1.) Fill in the empty spaces!

a) $\sum_{k=1}^{\infty} |w_k|$ converges ________ $\sum_{k=1}^{\infty} w_k$ converges (use one of "$\Leftarrow$", "$\Leftrightarrow$", "$\Rightarrow$").

b) The series $\sum_{k=1}^{\infty} w_k$ converges ________ for every $\varepsilon > 0$, there is a natural number N such that

$$|a_{n+1} + a_{n+2} + \ldots + a_{n+k}| < \varepsilon, \quad \text{whenever } n \geq N,\ k \geq 1.$$

(use one of "$\Leftarrow$", "$\Leftrightarrow$", "$\Rightarrow$").

c) The radius of converges for $\sum_{k=0}^{\infty} \frac{z^k}{k!}$ is ________________________________

d) Convergence of a power series on an open disk ________ absolute convergence on that disk. (use one of "$\Leftarrow$", "$\Leftrightarrow$", "$\Rightarrow$").

2.) For the principal branch of the logarithm, one finds the Taylor series

$$\mathrm{Log}(1+z) = \sum_{k=1}^{\infty} (-1)^{k-1} \frac{z^k}{k}.$$

For which z does this equality hold and what is remarkable about that?

For the solution, click on the following space:

3.) Why is the following statement false:
Any function can be represented by its Taylor series at $z_0 \in D$ throughout its domain.

For the solution, click on the following space:

4.) Write down two conditions each of which is sufficient to ensure that $f(z) = g(z)$ for all z in a domain D on which they are both holomorphic.

For some solutions, click on the following lines.

a) ______________________________

b) ______________________________

c)

____

d) ______________________________