

MA30056: Complex Analysis

SELF-ASSESSMENT SHEET 4: PATH INTEGRALS

- 1.) Evaluate $\int_{\gamma} \sin z \, dz$, where $\gamma(t) = i t^2$ with $t \in [1, 2]$.

For the solution, click on the following line:

$$\int_{\gamma} \sin z \, dz = \int_1^2 \underline{\hspace{15em}}$$

- 2.) Let f be a holomorphic function with continuous derivative on a domain D and γ be a closed regular path in D . What can you say about $\int_{\gamma} f'(z) \, dz$?

For the solution, click on the following space:

- 3.) Let $\gamma_1(t) = i t$ with $t \in [1, 4]$ and $\gamma_2(t) = i(t - 1)^2$ with $t \in [2, 3]$. What can you say about the relation between $\int_{\gamma_1} f \, dz$ and $\int_{\gamma_2} f \, dz$ for a continuous function f .

For the solution, click on the following space:

- 4.) Let $a, b \in \mathbb{C}$ be such that $|a| \geq |b|$, and let Γ be the straight line between a and b . Use the *ML*-inequality to show that

$$\left| \int_{\Gamma} z^2 \, dz \right| \leq |a|^2 |b - a|.$$

For the solution, click on the following space:

- 5.) To get some geometrical intuition of a function $f : \mathbb{C} \rightarrow \mathbb{C}$, you want to “visualize” it somehow. Give some methods to do so.

For the solution, click on the following lines:

(a) _____

(b) _____

(c) _____

(d) _____