MA30056: Complex Analysis

Self-Assessment Sheet 2: Paths, complex functions and complex differentiability

In the first two questions, click on "Evaluate" after you have ticked/filled in the appropriate statements/numbers.

1.) For each of the following Jordan curves, tick the property it has.

simple	○	○	○	○
closed	○	○	○	○

Evaluate

2.) If the functions f, g are holomorphic on $\mathbb{C}$, tick the functions that are in general also holomorphic on $\mathbb{C}$.

- ○ $(f(z))^3$.
- ○ $f(z) + g(z)$
- ○ $f(z)/g(z)$
- ○ $f(z^2 + 3i)$
- ○ $\operatorname{Re} f(z) + i \operatorname{Im} g(z)$
- ○ $5f(z) + ig(z)$

Evaluate

3.) Give the definition of domain and simply connected domain
For the solution, click on the following space:

Please turn over!

4.) Let $A \subset \mathbb{C}$ be an open set. Define a relation $\sim$ on A by $z_1 \sim z_2$ if there is a path from z_1 to z_2 in A. Indicate why $\sim$ is an equivalence relation, i.e., say why

(i) $z \sim z$ for all $z \in A$.
Click on the following line for the solution: ______________________

(ii) $z_1 \sim z_2 \Leftrightarrow z_2 \sim z_1$ for all $z_1, z_2 \in A$.
Click on the following line for the solution: ______________________

(iii) $z_1 \sim z_2$ and $z_2 \sim z_3 \Rightarrow z_1 \sim z_3$ for all $z_1, z_2, z_3 \in A$.
Click on the following line for the solution: ______________________

5.) Find the derivatives of the following functions:

- $f(z) = \left((z+i)^2 + (z-i)^2\right)^2$
  Click on the following line for the solution: ______________________
- $f(z) = \frac{a\,z+b}{c\,z+d}$
  Click on the following line for the solution: ______________________

6.) Show that $f(z) = \operatorname{Im} z$ is not differentiable anywhere.
For the solution, click on the following space:

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