MA30041: Metric Spaces

Self-Assessment Sheet 8: Contractions

1.) (i) Show that if $h \in C[a,b]$ (with the uniform metric) and $b - a < 1$ then $T : C[a,b] \to C[a,b]$ defined by

$$T(g)(x) = h(x) \int_a^x g(t)\,\mathrm{d}t$$

for $x \in [a,b]$, $g \in C[a,b]$ is a contraction mapping.
For a solution, click on the the following space:

(ii) Suppose further that h is differentiable and find the fixed point of T.
For a solution, click on the the following space:

2.) Let $f : [1,\infty) \to [1,\infty)$ be given by $f(x) = x + \frac{1}{x}$.

(i) Show that $|f(x) - f(y)| < |x - y|$ for each $x \neq y$.
For a solution, click on the the following space:

(ii) However, there is no x for which $x = f(x)$.
For a solution, click on the the following space:

3.) Let X be equipped with the discrete metric d. Show that every contraction mapping on (X,d) is a constant function.
For a solution, click on the the following space:
