

## MA30041: Metric Spaces

### SELF-ASSESSMENT SHEET 7: HOMEOMORPHISMS

- 1.) Let  $(X, d)$  and  $(Y, d')$  be metric spaces and let  $f : X \rightarrow Y$  be a surjective isometry. Prove that  $f$  is a homeomorphism.

*For a solution, click on the the following space:*

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- 2.) Show that if  $(X, d)$  is a discrete metric space and  $(Y, d')$  is any metric space, then any function  $f : X \rightarrow Y$  is continuous.

Use each of the (following) three equivalent criteria for continuity directly!

*For a solution, click on the the following spaces:*

- $\varepsilon$ - $\delta$  proof:
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- proof by sequences:
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- proof using open sets:
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- 3.) Let  $(X, d)$  and  $(Y, d')$  be discrete metric spaces. Show: Any bijective map  $f : X \rightarrow Y$  is a homeomorphism

*For a solution, click on the the following space:*

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- 4.) Show that  $\mathbb{R}$  and  $(0, \infty)$ , both equipped with the usual metric, are homeomorphic

*For a solution, click on the the following space:*

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