MA30041: Metric Spaces

Self-Assessment Sheet 6: Continuous functions

1.) Why is the function $f : \mathbb{R} \to \mathbb{R}^2$, $x \mapsto (\cos(x), \sin(x))$ continuous?
For a solution, click on the the following space:

2.) Given an example of a continuous function f from a metric space (X, d) to (Y, d') and a closed bounded subset $A \subset X$ s.t. $f(A)$ is neither closed nor bounded in (Y, d').
For a solution, click on the the following space:

3.) Recall the following definition from (M11):

> A sequence $(f_n) \subset C[a, b]$ converges *pointwise* to $f : [a, b] \to \mathbb{R}$ if $\forall t \in [a, b]$ and $\forall \varepsilon > 0$ $\exists N \in \mathbb{N}$ s.t. $\forall m \geq N$ we have $|f_m(t) - f(t)| < \varepsilon$.

What, by analogy, must be the corresponding definition for a $f : X \to Y$ from a metric space (X, d) to (Y, d').
For a solution, click on the the following space:

4.) Recall the following definition from (M11):

> A sequence $(f_n) \subset C[a, b]$ converges *uniformly* to $f : [a, b] \to \mathbb{R}$ if $\forall \varepsilon > 0$ $\exists N \in \mathbb{N}$ s.t. $\forall m \geq N$ and $\forall t \in [a, b]$ we have $|f_m(t) - f(t)| < \varepsilon$.

What, by analogy, must be the corresponding definition for a $f : X \to Y$ from a metric space (X, d) to (Y, d').
For a solution, click on the the following space:
