MA30041: Metric Spaces

Self-Assessment Sheet 5: Topology of metric spaces

1.) Show: For any two points x, y of a metric space (X, d), there exist disjoint open balls s.t. one is centred at x and the other centred at y.
For a solution, click on the the following space:

2.) Give an example of a metric space (X, d) s.t. X contains a set U which is both open and closed and a set V which is neither open nor closed.
For a solution, click on the the following space:

3.) Given an example a subset A of of $\mathbb{R}$ (equipped with the usual metric) s.t. $\operatorname{diam}(\operatorname{int} A) < \operatorname{diam} A$.
For a solution, click on the the following space:

4.) Show: $X = \operatorname{intA} \cup \partial A \cup \operatorname{int}(A^c)$ for any subset A of a metric space (X, d).
For a solution, click on the the following space:
(note, we are using the notation $A_1 - A_2$ for $A_1 \setminus A_2$ here)

5.) Can you find a metric spaces (X, d) where

(i) $[0, 1]$ is clopen?

(ii) $(0, \frac{1}{2})$ is clopen?

(iii) $[0, \frac{1}{2})$ is open but not closed?

For a solution, click on the the following space:
