MA30041: Metric Spaces

SELF-ASSESSMENT SHEET 1: EXAMPLES OF METRIC SPACES

1.) Check that the maximum metric d_∞ on $\mathbb{R}^n$, given by $d_\infty(x,y) = \max_{1\le i\le n} |x_i - y_i|$ for $x = (x_1,\dots,x_n), y = (y_1,\dots,y_n) \in \mathbb{R}^n$, is indeed a metric.
For hints, click on the following lines:

(MS1) ______________________________

(MS2) ______________________________

(MS3) ______________________________

(MS4) ______________________________

2.) Check that the sum metric d_1 on $\mathbb{R}^n$, given by $d_1(x,y) = \sum_{i=1}^n |x_i - y_i|$ for $x = (x_1,\dots,x_n), y = (y_1,\dots,y_n) \in \mathbb{R}^n$, is indeed a metric.
For hints, click on the following lines:

(MS1) ______________________________

(MS2) ______________________________

(MS3) ______________________________

(MS4) ______________________________

3.) For the maximum metric d_∞, the Euclidean metric d_2 (respectively, d_{E}) and the sum metric d_1 on $\mathbb{R}^n$ show:

$$d_\infty(x,y) \le d_2(x,y) \le d_1(x,y) \quad \forall x,y \in \mathbb{R}^n.$$

For hints, click on the following lines:

- "$d_\infty \le d_2$": ______________________________
- "$d_2 \le d_1$": ______________________________

Please turn over!

4.) Check that the *jungle river metric* or *barbed wire metric* on $\mathbb{R}^2$ is a metric. Recall that this metric is given by

$$d\left((x_1, x_2), (y_1, y_2)\right) = \begin{cases} |x_2 - y_2| & \text{if } x_1 = y_1, \\ |x_1 - y_1| + |x_2| + |y_2| & \text{if } x_1 \neq y_1. \end{cases}$$

For hints, click on the following lines:

(MS1) ______________________________

(MS2) "$\Leftarrow$": ______________________________

"$\Rightarrow$": ______________________________

(MS3) ______________________________

(MS4) ______________________________