MA30041: Metric Spaces

OLD EXAMS 9: CONNECTEDNESS

1.) *From the 2004/05-exam:*

(a) Let (X, d) be a metric space and $A \subset X$.

(i) What is meant by a *path* in (X, d) joining x to y?

(ii) When is $A \subset X$ said to be *connected*?

(iii) When is $A \subset X$ said to be *path-connected*?

(iv) Does connectedness imply path-connectedness, or does path-connectedness imply connectedness. Are they in fact equivalent?

(b) Suppose that A is a connected subset of (X, d) and that f is a continuous function from (X, d) to $(Y, \tilde{d})$. Show that $f(A)$ is connected in $(Y, \tilde{d})$.
[*State explicitly how you characterize continuity of functions between metric spaces.*]

(c) Let $(C[0, 1], d_{\max})$ denote the usual metric space of continuous functions $f : [0, 1] \to \mathbb{R}$ with

$$d_{\max}(f, g) = \max_{x \in [0,1]} |f(x) - g(x)|, \qquad f, g \in C[0, 1].$$

Let $f_0, g_0 \in C[0, 1]$ be functions with $f_0(x) \leq g_0(x)$ for all $x \in [0, 1]$ and let

$$A = \{f \in C[0, 1] \mid f_0(x) \leq f(x) \leq g_0(x), x \in [0, 1]\}.$$

Show that A is path-connected in $(C[0, 1], d_{\max})$.