MA30041: Metric Spaces

OLD EXAMS 8: COMPACTNESS

1.) *From the 2006/07-exam:*

(a) (i) When is a set K in a metric space (X, d) sequentially compact?

(ii) Suppose that K is sequentially compact in (X, d), that $F \subset K$ and F is closed in (X, d). Show that F is sequentially compact in (X, d).

(b) (i) What is a homeomorphism between metric spaces (X, d) and $(Y, \tilde{d})$?

(ii) Suppose that (X, d) is sequentially compact and that $h : X \to Y$ is a continuous bijection from (X, d) to $(Y, \tilde{d})$. Without giving proofs, make a careful list of statements that lead to the conclusion that h is a homeomorphism from (X, d) to $(Y, \tilde{d})$.

[*State explicitly how you characterize continuity of functions between metric spaces.*]

(c) Let (X, d) denote $(0, \infty)$ with the standard metric. Let $Y = (0, 1) \subset X$. Write down a homeomorphism between (X, d) and $(Y, d|_Y)$.