

MA30041: Metric Spaces

OLD EXAMS 6: HOMEOMORPHISMS

1.) *From the 2002/03-exam:*

Let (X, d) , (\tilde{X}, \tilde{d}) be metric spaces.

(a) What is meant by saying that

(i) (X, d) and (\tilde{X}, \tilde{d}) are *homeomorphic* spaces,

(ii) (X, d) and (\tilde{X}, \tilde{d}) are *isometric* spaces?

(b) Show that if (X, d) and (\tilde{X}, \tilde{d}) are isometric spaces then they are homeomorphic spaces.

(c) Suppose that (X, d) and (\tilde{X}, \tilde{d}) are homeomorphic spaces. If (X, d) is complete does it necessarily follow that (\tilde{X}, \tilde{d}) is complete? What if we assume that (X, d) and (\tilde{X}, \tilde{d}) are isometric spaces? (Give a proof or a counterexample in each case.)

2.) *From the 2005/06-exam:*

(b) Suppose that (X, d) is a metric space. Let $\tilde{d}(x, y) = d(x, y)/(1 + d(x, y))$ for all $x, y \in X$. Assuming that \tilde{d} is a metric on X , show that the metric spaces (X, d) and (X, \tilde{d}) are homeomorphic.