1.) *From the 2004/05-exam:*

(a) Let (X, d) be a metric space. Give precise definitions of the following.

(i) $B_r(x)$ where $x \in X$ and $r > 0$.

(ii) int A, where $A \subset X$.

(iii) A *limit point* of $A \subset X$.

(iv) cl A, where $A \subset X$.

(v) A *closed* set in (X, d).

(vi) An *open* set in (X, d).

(b) Let (X, d) be a metric space. Show that, for every $A \subset X$, $(\text{int } A)^c = \text{cl}\,(A^c)$ and deduce that the complement of every open set is a closed set.

2.) *From the 2006/07-exam:*

(i) What is an *interior point* of a set A in a metric space (X, d)?

(ii) What is a *limit point* of a set A in a metric space (X, d)?

(iii) Denote the set of limit points of A by A' and the set of interior points of A by int A. Show that, for any set $U \subset X$, int $U = U \setminus (U^c)'$.

(iv) Deduce that a set U is open in (X, d) if it contains no limit points of its complement.