MA30041: Metric Spaces

OLD EXAMS 3: CAUCHY SEQUENCES & COMPLETENESS

1.) *From the 2003/04-exam:*

Prove that if $C[0,1]$ is endowed with the supremum (or uniform) metric

$$d(f,g) = \sup_{t\in[0,1]} |f(t) - g(t)| \qquad \forall f, g \in C[0,1],$$

then $(C[0,1], d)$ is a complete metric space.

2.) *From the 2006/07-exam:*

(a) (i) State the triangle-inequality axiom for a metric space (X, d)?

(ii) Suppose that $x, y \in X$ where (X, d) is a metric space and $d(x, y) = 2r$. Prove that $B_r(x) \cap B_r(y) = \varnothing$.

(b) Suppose that (X, d) is a metric space and let

$$\tilde{d}(x,y) = \frac{d(x,y)}{1 + d(x,y)}, \qquad \forall x, y, \in X.$$

Show that $\tilde{d}$ satisfies the triangle-inequality axiom for a metric space.