

MA30041: Metric Spaces

OLD EXAMS 2: SEQUENCES

1.) *From the 2004/05-exam:*

- (a) In a metric space (X, d) , what is meant by
 - (i) a *convergent* sequence,
 - (ii) a *bounded* sequence,
 - (iii) a *Cauchy* sequence?
- (b) Show that
 - (i) every convergent sequence is Cauchy;
 - (ii) every Cauchy sequence is bounded.

2.) *From the 2006/07-exam:*

- (i) What is a Cauchy sequence in a metric space (X, d) ?
- (ii) Give the definition of a complete metric space.
- (iii) Show, from first principles, that a metric space is complete iff every Cauchy sequence has a convergent subsequence.