1.) *From the 2004/05-exam:*

(i) What is a metric space?

(ii) What is a discrete metric space?

(iii) If (X, d) is a metric space, show that

$$|d(x, y) - d(z, y)| \leq d(x, y) \qquad \forall x, y, z \in X. \quad (1)$$

(iv) If (X, d) is a discrete metric space, give necessary and sufficient conditions on x, y, z for equality to hold in (1).

2.) *From the 2007/08-exam:*

(a) What is a metric subspace $(X, d|_X)$ of a metric space (X, d)?

(b) Which, if any, of the following is a metric space?

(i) The set $\mathbb{C}$ of complex numbers with $d(z_1, z_2) = ||z_1| - |z_2||$ for $z_1, z_2 \in \mathbb{C}$.

(ii) The set L^+ of all lines, through the origin in the plane, with positive slope where

$$d\left(\ell(m_1), \ell(m_2)\right) = \left|\frac{1}{m_1} - \frac{1}{m_2}\right|.$$

Here, elements of L^+ are denoted by $\ell(m) = \{(x, mx) \mid x \in \mathbb{R}\} \subset \mathbb{R}^2$, $m > 0$.