MA10103: Foundation Mathematics I

PROBLEM SHEET 8

Please, do all questions and hand in solutions to the starred questions at the lecture on *Monday 26th November*.

1. Remember that

$$\sin 45^\circ = 1/\sqrt{2}; \quad \cos 45^\circ = 1/\sqrt{2}; \quad \sin 30^\circ = 1/2; \quad \cos 30^\circ = \sqrt{3}/2.$$

Remember also that

$$\sin(A+B) = \sin A \cos B + \cos A \sin B \quad \text{and} \quad \cos(A+B) = \cos A \cos B - \sin A \sin B.$$

Find, without using a calculator:

$$\sin 15^\circ; \quad \cos 15^\circ; \quad \sin 75^\circ; \quad \cos 315^\circ; \quad \tan 75^\circ; \quad \sin 135^\circ;$$
$$\tan 105^\circ; \quad \sin^2 30^\circ + \cos^2 30^\circ; \quad \tan^2 45^\circ - \tfrac{1}{\cos^2 45^\circ}; \quad \tan 345^\circ.$$

2.* By completing the square, find the highest point on the curve $y = 16 - 6x - x^2$, and find the x- and y-intercepts. Sketch the curve, marking the points you have found.

3. By completing the square, find the lowest point on the curve $y = x^2 + 4x + 6$, and find the x- and y-intercepts. Do the same for $y = x^2 + 4x + 4$. Sketch both curves.

4.* (a) Sketch the curve $y = x$.
   (b) Sketch the curve $y = \frac{2}{x}$, marking all asymptotes.
   (c) Sketch the curve $y = x + \frac{2}{x}$ marking all asymptotes and x- and y-intersections.

5. Sketch the curve $y = \dfrac{1}{x-2}$, marking all asymptotes and x- and y- intercepts.

6. Sketch the curve $y = \sin x$. Where are the x- and y-intercepts? Does it have asymptotes? Is it an even or an odd function (or neither even nor odd)?

7. *Repetition question:* Solve the following equations:

   (a) $\log x = 2\log(x-1)$.
   (b) $\sin^2\theta + 3\cos\theta - \frac{1}{2} = 0$.
   Give the solution for θ in degrees and correct to 2 decimal places.