**MA10103: Foundation Mathematics I**

Problem Sheet 4 – Assessed coursework (worth 12.5% of final mark)

Please do **ALL** of the following questions.

Please hand in your work at the Mathematics Reception Desk (1West 3.12) not later than Monday 29th October at 12.15pm. You have to sign your work in and it should be accompanied by a signed coursework cover sheet (distributed with this assignment and also available from the reception desk.)

**The work which you hand in should be the result of your own efforts and you should not collaborate with anyone else in this assignment. You should be prepared to explain anything which you write to the lecturer if asked to do so. Students caught cheating will be penalised.**

1. Factorise the following, using integers, if you can.

$$x^2 - 81; \quad 10x^2 + 9x + 2; \quad y^2 - 2yr + r^2; \quad x^2 - x - 3; \quad 1 - x - 6x^2.$$

   If any factorisation cannot be done, say so.

2. Express in partial fractions:

$$\frac{2x+1}{(2x-1)(x+1)}; \quad \frac{4}{x^2-2x-3}.$$

3. Simplify the following fractions (by rationalising the denominator):

$$\frac{\sqrt{5}}{5-\sqrt{5}}; \quad \frac{1}{\sqrt{13}+\sqrt{7}}; \quad \frac{3-\sqrt{3}}{3+\sqrt{3}}.$$

4. Simplify the following:

$$9^{1/2} \times 3^{-3}; \quad \frac{x^{-1/3} \times x}{x^{-2/3}}; \quad \left(z^3\right)^{5/6} \times z^{-1/2}; \quad \left(\sqrt{t^{1/2}}\right)^4 \times t^{-1}$$

5. Rewrite the following as equations involving logarithms:

$$5^3 = 125; \quad 7^{-2} = \frac{1}{49}; \quad d^w = k; \quad 36^{1/2} = 6.$$

*Please turn over!*

6. Express the following in terms of $\log p$ and $\log q$:

$$\log(p^2\, q); \quad \log(p/q^2).$$

Write each of the following as single logarithm:

$$\log p + 5\log q; \quad \log q - 3\log p; \quad m\log p - (\log q)/m.$$

7. Solve the equation $5^x = 7$ for $x$. Also, write the result correct to 5 decimal places.

8. The pH-value of a liquid solution measures the acidity or alkalinity (or basicity). It is calculated as follows: given the concentration of $H^+$-ions (more precisely, of $H_3O^+$-ions) in mol per litre, one has

$$\text{pH} = -\log_{10}(\text{concentration of } H^+\text{-ions in mol/l}).$$

Pure water has a pH-value of 7. A solution is called acid, if the pH-value is below 7. If it is above 7, the solution is called basic.

(a) Tea has approximately a concentration of $10^{-5.5}$mol/l of $H^+$-ions. Calculate the pH-value.

(b) The pH-value of orange juice is approximately 3.5. What is the corresponding concentration of $H^+$-ions?

(c) In example (a), we add lemon juice to the tea. By doing this, we double the concentration of $H^+$-ions. What is the pH-value now?