MA10103: Foundation Mathematics I

PROBLEM SHEET 11

Revision lectures: Monday 7 January 2007 at 9.15 & Tuesday 8 January 2007 at 10.15. In the revision lecture on 7 January, solutions of the starred questions will be given. For the lecture on 8 January, please email problems (e.g., from a previous or this problem sheet, from the lecture or from previous exam papers) you would like to discus.

1.* Find $f'(x)$ in the following cases.

$$f(x) = \mathrm{e}^{-3\,x}; \quad f(x) = \sin x\ \cos x; \quad f(x) = x^2\ \log x;$$

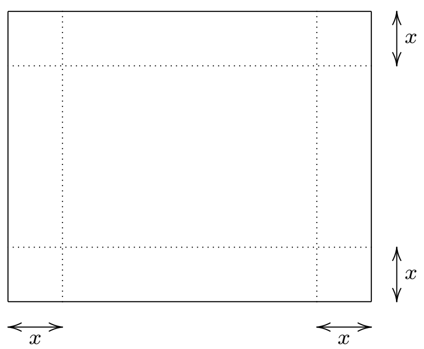
$$f(x) = \sin(x^3+2); \quad f(x) = \ln\left(\sin(x^3+2)\right).$$

2.* For each function, find the x and y coordinates of the stationary points and determine whether each stationary point is a maximum or a minimum.

$$y = \ln(1+x^2); \qquad y = (x+1)^2; \qquad y = x - e^x; \qquad y = x^3 - 3x^2 - 9x + 1.$$

3.* An open rectangular box is to be made from a metre square of cardboard, by cutting equal squares (of side x say) away from the corners, and then folding parallel to the edges. Find the value of x for the box of greatest volume.

[Hint: write down an expression for the volume V as a function of x, then minimise.]



4.* A particle whose position at time t is given by the coordinates $x = \frac{1}{\sqrt{2}}\cos t$ and $y = \sqrt{2}\sin t$, travels along a curved path in the plane from time $t = 0$ to times $t = 2\,\pi$.

(a) Show that $\frac{\mathrm{d}}{\mathrm{d}t}\,(x^2+y^2) = 3\ \sin t\ \cos t$.

(b) Hence find the distance of the particle from the origin when it is at its closest point to the origin.

5.* Air is being pumped into a spherical balloon of radius 10 cm. If the radius of the balloon is increasing at 3 cm/min, how fast is the air being pumped in?

Please turn over!

Here are "long questions" taken from previous exams ("Section B").

6. Sketch the curve $y = \dfrac{x}{1-x^2}$, marking any stationary points, intersections with the axes, and asymptotes.

7. A rectangular wooden window frame of width w and height h is to be made. Write down expressions for
   (i) the total length of wood which will be used to make the frame,
   and
   (ii) the area of the window enclosed by the frame,
   as functions of w and h. (In your formulae you may neglect the thickness of the wood.)

   (a) If the total length of the wood is fixed at 1 metre, how should w and h be chosen so that the area of the frame is maximised?

   (b) If the area of the window is fixed at 2 square metres, how should w and h be chosen in order that the total length of wood used is minimised?

   In each case, justify your results.

8. A rectangular window frame is to be made, to enclose a space of area 1 square metre. The wood for the vertical sides costs four times as much per metre as that for the horizontal sides. Find the dimensions for the frame that minimise the total cost.

9. A fully closed cylindrical tin can with circular cross-section is to be made from 2π square units of metal, to enclose the greatest possible volume. Find the maximum value of the volume and the corresponding radius and height of the can. Explain your reasoning.

Merry Christmas and Best Wishes for the Coming Year! Good Luck in the Final Exam!