MA10103: Foundation Mathematics I

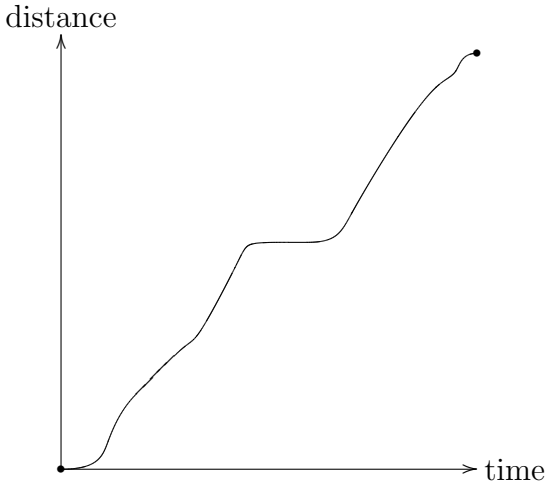
Problem Sheet 10

Please, do all questions and hand in solutions to the starred questions at the lecture on *Monday 10th December*.

1. To the right, you find a schematic diagram that gives the distance travelled (say, by a car) as function of the time. It is a graph of some function.

   (a) Sketch (also schematically) the derivative. Here, the derivative has a physical meaning: it is the "change of distance" and therefore the velocity (as function of the time).

   (b) Also sketch the derivative of the derivative (also called the *second derivative*). This is the "change of velocity" and therefore the acceleration as function of the time. If this function is negative, the car brakes; if the driver puts her foot on the gas, this function is positive.



2. Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ in each of the following cases:

$$y = \frac{1}{x+1}; \qquad y = (\sin x)\,(x+2); \qquad y = \sin(2\,x+3);$$
$$y = (x^3+2)^6; \qquad y = x + (3-x)^5; \qquad y = \frac{\cos(2\,x)}{x^2};$$
$$y = x^3\,\cos(3\,x); \qquad y = \sqrt{1+2\,x}; \qquad y = \sqrt{1+x^2}.$$

3.* Let $f(x) = (x^2+1)^{10}$. Find $f'(x)$ and $f'(1)$.

4.* Let $g(x) = \sin\left((x^2+1)^9\right)$. Find $g'(x)$.

5. Find the lowest point on the curve $y = 3\,x^2 + 4\,x - 3$ by

   (a) completing the square.

   (b) using the derivative.

Please turn over!

6.* Consider the function $f(x) = x^3 + 3\,x^2 + x$.

(a) Calculate its derivative $f'(x)$. Sketch $y = f'(x)$; for this, calculate its stationary point(s), x- and y-intercepts.

(b) Using your calculations in (a), find the stationary points of $y = f(x)$. For which values of x is $y = f(x)$ increasing or decreasing. Also state whether the stationary points are minima or maxima.

(c) Sketch $y = f(x)$ for $-3 \leq x \leq 1$. To do this, not only mark the stationary points you have calculated in (b), but also calculate the y-intercept and make a table

x	-3	-2.5	-2	$\cdots$
$y = f(x)$	-3	0.625	$\cdots$	$\cdots$

Try to factorise $x^3+3\,x^2+x$ in order to find the exact values of the x-intercepts.

7. Sketch the curve $y = \tan x$. Where are the asymptotes and x-intercepts? Also sketch the derivative of $\tan x$, i.e., sketch $y = 1/\cos^2 x$.