MA10103: Foundation Mathematics I

Solutions of Problem Sheet 7

1. $(1,6)$ and $(3,5)$: distance $\sqrt{2^2+1^2}=\sqrt{5}=2.236$ (3 d.p.), midpoint $(2,5.5)$;
$(4,8)$ and $(-3,-4)$: distance $\sqrt{7^2+12^2}=\sqrt{193}=13.892$ (3 d.p.), midpoint $(0.5,2)$;
$(2,2)$ and $(7,14)$: distance $\sqrt{5^2+12^2}=\sqrt{169}=13$, midpoint $(4.5,8)$.

2*. (a) $y=3x-5$
(b) $y=\frac{1}{2}x+\frac{7}{2}$
(c) $y=-\frac{1}{3}x+5\frac{1}{3}$ or $y=-\frac{1}{3}x+\frac{16}{3}$.
(d) $y=-\frac{3}{2}x$ or $3x+2y=0$.

3*. $y=3x+2$ and $y=7x-6$: substract to get $4x-8=0$, hence $x=2$ and therefore $y=8$. The point of intersection is $(2,8)$.
$y=-3x-4$ and $y=2x+3$: substract to get $5x+7=0$, hence $x=-\frac{7}{5}$ and therefore $y=\frac{1}{5}$. The point of intersection is $\left(-\frac{7}{5},\frac{1}{5}\right)$.

4. Substitute $y=x+2$ in $x^2+y^2=4$ to get $x^2+(x+2)^2=4$, hence $2x^2+4x=0$. Noting that $2x^2+4x=2(x+2)x$, one gets the solutions $x=0$ (then $y=2$) and $x=-2$ (then $y=0$). Thus the points of intersection are $(0,2)$ and $(-2,0)$.

5. (a) $U(5,1)$, $V(6.5,5)$ and $W(1.5,4)$.
(b) $f:\ y=-\frac{7}{2}x+\frac{37}{2}$; $\quad g:\ y=\frac{10}{13}x$; $\quad h:\ y=-\frac{4}{17}x+\frac{74}{17}$.
(c) One point for all three cases: $G(\frac{13}{3},\frac{10}{3})$.
This point is called the *centroid* or *centre of gravity*. Why?
(d)
- $|GU|=\sqrt{\left(\frac{2}{3}\right)^2+\left(\frac{7}{3}\right)^2}=\frac{\sqrt{53}}{3}=2.427$ (3 d.p.) and $|GR|=\sqrt{\left(\frac{4}{3}\right)^2+\left(\frac{14}{3}\right)^2}=\frac{2\sqrt{53}}{3}=4.853$ (3 d.p.).
- $|GV|=\sqrt{\left(\frac{13}{6}\right)^2+\left(\frac{5}{3}\right)^2}=\frac{\sqrt{269}}{6}=2.734$ (3 d.p.) and $|GP|=\sqrt{\left(\frac{13}{3}\right)^2+\left(\frac{10}{3}\right)^2}=\frac{\sqrt{269}}{3}=5.467$ (3 d.p.).
- $|GW|=\sqrt{\left(\frac{17}{6}\right)^2+\left(\frac{2}{3}\right)^2}=\frac{\sqrt{305}}{6}=2.911$ (3 d.p.) and $|GQ|=\sqrt{\left(\frac{17}{3}\right)^2+\left(\frac{4}{3}\right)^2}=\frac{\sqrt{305}}{3}=5.821$ (3 d.p.).

The centre of gravity G divides the line joining a corner and the midpoint of the opposite side in the ratio $2:1$. This holds in every triangle!

Please turn over!

6. (a) £ $550 \times 1.05 \times 1.05 \times 1.05 \times 1.05 \times 1.05 =$ £ $550 \times 1.05^5 =$ £ 701.95 (2 d.p.).

(b) $1.05^x = 2 : \ x = \frac{\log 2}{\log 1.05} = 14.207$ (3 d.p.), so after 15 years the money has (more than) doubled.
$1.05^y = 3 : \ y = \frac{\log 3}{\log 1.05} = 22.517$ (3 d.p.), so after 23 years the money has (more than) tripled.