MA10103: Foundation Mathematics I

Solutions of Problem Sheet 4 (assessed coursework)

1. $x^2 - 81 = (x-9)(x+9)$ — 1 Point
   $10x^2 + 9x + 2 = (5x+2)(2x+1)$ — 1 Point
   $y^2 - 2yr + r^2 = (y-r)^2$ — 1 Point
   $x^2 - x - 3$ factorisation using integer coefficeints not possible — 1 Point
   Factorisation using reals (surds) is $x^2-x-3=\left(x-\frac{1+\sqrt{13}}{2}\right)\left(x-\frac{1-\sqrt{13}}{2}\right)$
   $1 - x - 6x^2 = (-3x+1)(2x+1) \overset{\text{or}}{=} (3x-1)(-2x-1)$ — 1 Point

2. $\frac{2x+1}{(2x-1)(x+1)} = \frac{A}{2x-1} + \frac{B}{x+1} = \frac{A(x+1)+B(2x-1)}{(2x-1)(x+1)}$ — $\frac{1}{2}$ Point
   Equat numerators: $2x + 1 = A(x+1) + B(2x-1)$ — $\frac{1}{2}$ Point
   At $x = -1$: $-1 = B \times (-3)$, hence $B = \frac{1}{3}$. — $\frac{1}{2}$ Point
   At $x = \frac{1}{2}$: $2 = A \times \frac{3}{2}$, hence $A = \frac{4}{3}$. — $\frac{1}{2}$ Point
   So, $\frac{2x+1}{(2x-1)(x+1)} = \frac{4}{3\times(2x-1)} + \frac{1}{3\times(x+1)}$. — $\frac{1}{2}$ Point

   $\frac{4}{x^2-2x-3} = \frac{4}{(x-3)(x+1)} = \frac{A}{x-3} + \frac{B}{x+1} = \frac{A(x+1)+B(x-3)}{(x-3)(x+1)}$ — $1\frac{1}{2}$ Points
   Equat numerators: $4 = A(x+1) + B(x-3)$ — $\frac{1}{2}$ Point
   At $x = -1$: $4 = B \times (-4)$, hence $B = -1$. — $\frac{1}{2}$ Point
   At $x = 3$: $4 = A \times 4$, hence $A = 1$. — $\frac{1}{2}$ Point
   So, $\frac{4}{x^2-2x-3} = \frac{1}{x-3} - \frac{1}{x+1}$. — $\frac{1}{2}$ Point

3. $\frac{\sqrt{5}}{5-\sqrt{5}} = \frac{\sqrt{5}\times(5+\sqrt{5})}{(5-\sqrt{5})(5+\sqrt{5})} = \frac{5\sqrt{5}+5}{20} = \frac{\sqrt{5}+1}{4}$ — 1 Point
   $\frac{1}{\sqrt{13}+\sqrt{7}} = \frac{\sqrt{13}-\sqrt{7}}{(\sqrt{13}+\sqrt{7})(\sqrt{13}-\sqrt{7})} = \frac{\sqrt{13}-\sqrt{7}}{6}$ — 1 Point
   $\frac{3-\sqrt{3}}{3+\sqrt{3}} = \frac{(3-\sqrt{3})^2}{(3+\sqrt{3})(3-\sqrt{3})} = \frac{9-6\sqrt{3}+3}{6} = 2 - \sqrt{3}$ — 1 Point

4. $9^{1/2} \times 3^{-3} = \sqrt{9} \times 3^{-3} = 3 \times 3^{-3} = 3^{-2} = \frac{1}{3^2} = \frac{1}{9}$ — 1 Point
   $\frac{x^{-1/3}\times x}{x^{-2/3}} = x^{-\frac{1}{3}+1-(-\frac{2}{3})} = x^{4/3}$ — 1 Point
   $\left(z^3\right)^{5/6} \times z^{-1/2} = z^{5/2} \times z^{-1/2} = z^2$ — 1 Point
   $\left(\sqrt{t^{1/2}}\right)^4 \times t^{-1} = t^{\frac{1}{2}\times\frac{1}{2}\times 4} \times t^{-1} = t^{1-1} = t^0 = 1$ — 1 Point

Please turn over!

5. $5^3 = 125: \quad \log_5 125 = 3$ 1 Point
$7^{-2} = \frac{1}{49}: \quad \log_7 \frac{1}{49} = -2$ 1 Point
$d^w = k: \quad \log_d k = w$ 1 Point
$36^{1/2} = 6: \quad \log_{36} 6 = \frac{1}{2}$ 1 Point

6. $\log(p^2\, q) = \log(p^2) + \log q = 2\log p + \log q$ 1 Point
$\log(p/q^2) = \log p - \log(q^2) = \log p - 2\log q$ 1 Point
$\log p + 5\log q = \log p + \log(q^5) = \log(p\, q^5)$ 1 Point
$\log q - 3\log p = \log q - \log(p^3) = \log(q/p^3)$ 1 Point
$m\log p - (\log q)/m = \log(p^m) - \frac{1}{m}\log q = \log(p^m) - \log\left(q^{1/m}\right)$
$= \log(p^m/q^{1/m}) \overset{\text{or}}{=} \log(p^m/\sqrt[m]{q})$ $1\frac{1}{2}$ Points

7. $5^x = 7: \quad x = \log_5 7$ 1 Point
$x = \log_5 7 = \log_{10} 7/\log_{10} 5 = 1.20906$ (5 d.p.) $1\frac{1}{2}$ Points

8. (a) $-\log_{10} 10^{-5.5} = -(-5.5) = 5.5$. So the pH-value of tea is 5.5. 1 Point

(b) $3.5 = -\log_{10} x$, so $-3.5 = \log_{10} x$ and therefore $x = 10^{-3.5}$. So the concentration of H^+-ions in orange juice is $10^{-3.5}$ mol/l. 1 Point

(c) $-\log_{10}(2 \times 10^{-5.5}) = -(\log_{10} 2 + \log_{10} 10^{-5.5}) = 5.5 - \log_{10} 2 \approx 5.5 - 0.3 = 5.2$. The pH-value is now 5.2. 1 Point

Note: Whatever the pH-value is, if the concentration of H^+-ions is doubled, the pH-value is lowered by 0.3!

Total: 33 Points