MA10103: Foundation Mathematics I

Solutions of Problem Sheet 3

1. $\frac{3^4}{3^2\times 9^3} \overset{R3}{=} \frac{3^4}{3^2\times 3^6} \overset{R1}{=} \frac{3^4}{3^8} \overset{R2}{=} \frac{1}{3^4} = \frac{1}{81}$; $8^{1/2}\times 2^{-3} \overset{R3}{=} \frac{2^{\frac{3}{2}}}{2^3} \overset{R2}{=} 2^{-\frac{3}{2}} \overset{R4}{=} \frac{1}{\sqrt{8}} = \frac{\sqrt{2}}{4}$; $(2^3)^{1/2}\times 4^{1/4} \overset{R3}{=} 2^{\frac{3}{2}} \times 2^{\frac{2}{4}} \overset{R1}{=} 2^2 = 4$; $\frac{x^{1/4}\times x^{5/4}}{x^{-1/6}} \overset{R1}{=} \frac{x^{\frac{6}{4}}}{x^{-\frac{1}{6}}} \overset{R2}{=} x^{\frac{10}{6}} = x^{\frac{5}{3}} = (\sqrt[3]{x})^5$; $\frac{p^{1/2}\times p^{-3/4}}{p^{-5/4}} \overset{R1}{=} \frac{p^{-\frac{1}{4}}}{p^{-\frac{5}{4}}} \overset{R2}{=} p$; $\left(\sqrt{t}\right)^5 \times \left(\sqrt{t^3}\right) \overset{R3}{=} t^{\frac{5}{2}} \times t^{\frac{3}{2}} \overset{R1}{=} t^4$; $(y^2)^{3/2} \times y^{-3} \overset{R3}{=} y^3 \times y^{-3} \overset{R1}{=} y^0 = 1$; $\frac{(16)^{5/4}}{8^{4/3}} \overset{R3}{=} \frac{2^{4\times\frac{5}{4}}}{2^{3\times\frac{4}{3}}} = \frac{2^5}{2^4} \overset{R2}{=} 2$; $\frac{y^{1/2}}{y^{-3/4}} \times \sqrt{y^{1/2}} \overset{R4}{=} \frac{y^{\frac{1}{2}}\times y^{\frac{1}{4}}}{y^{-\frac{3}{4}}} \overset{R1}{=} \frac{y^{\frac{3}{4}}}{y^{-\frac{3}{4}}} \overset{R2}{=} y^{\frac{3}{2}}$; $x^{(5/2)} \times x^2/x^{1/2} \overset{R1}{=} \frac{x^{\frac{9}{2}}}{x^{\frac{1}{2}}} \overset{R2}{=} x^4$.

2*. $\left(\frac{1}{3}\right)^{-1} = 3^1 = 3$; $\left(\frac{1}{4}\right)^{5/2} \overset{R3}{=} \left(\sqrt{\frac{1}{4}}\right)^5 = \frac{1}{2^5} = \frac{1}{32}$; $(8)^{-2/3} \overset{R3}{=} \left(\sqrt[3]{8}\right)^{-2} = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$; $\frac{1}{(16)^{-1/2}} = \sqrt{16} = 4$; $\left(\frac{1}{9}\right)^{-3/2} = (3^{-2})^{-\frac{3}{2}} \overset{R3}{=} 3^3 = 27$.

3*. (a) $\frac{18\times 10^{30}\text{m}}{3\times 10^8 \text{m s}^{-1}} = 6\times 10^{22}\text{s}$ (that would be 2×10^{15} years – the age of the universe is believed to be 1.4×10^{10} years)
(b) $(60 \times 60)\text{s} \times 3 \times 10^8 \text{m s}^{-1} = 1.08 \times 10^{12}\text{m} = 1.08 \times 10^9\text{km}$ (the distance Sun — Uranus (Uranus was discovered by W. Herschel in 1781 while living in Bath [Herschel, not Uranus ;-)]) is approx. 2.9×10^9km; 1.08×10^9km is approx. halfway between Jupiter and Saturn)
(c) $\frac{81\times 10^9\text{km}}{3\times 10^8\text{m s}^{-1}} = \frac{81\times 10^{12}\text{m}}{3\times 10^8\text{m s}^{-1}} = 2.7 \times 10^5\text{s} = 75$ hours $= 3$ days 3 hours

4. Total distance: $2\times 20\text{km} = 40\text{km}$; total time for journey: $\frac{20\text{km}}{30\text{km/h}} + \frac{20\text{km}}{20\text{km/h}} = \frac{2}{3}\text{h} + 1\text{h} = \frac{5}{3}\text{h} = 100\text{min}$; average speed $\frac{40\text{km}}{\frac{5}{3}\text{h}} = 24\text{km/h}$
20km ≈ 12.4mi; 30km/h ≈ 18.6mph; 20km/h ≈ 12.4mph; 24km/h ≈ 14.9mph

5*. $10^4 = 10000 : \log_{10} 10000 = 4$; $3^2 = 9 : \log_3 9 = 2$; $10^{-2} = 0.01 : \log_{10} 0.01 = -2$; $4^{1/2} = 2 : \log_4 2 = \frac{1}{2}$; $p = q^4 : \log_q p = 4$.
$\log_{10} 100000 = 5 : 10^5 = 100000$; $\log_2 8 = 3 : 2^3 = 8$; $\log_5 1 = 0 : 5^0 = 1$; $\log_3 27 = 3 : 3^3 = 27$; $\log_x y = z : x^z = y$.

6. $\log pq \overset{R1}{=} \log p + \log q$; $\log pqr \overset{R1}{=} \log p + \log q + \log r$; $\log(p/q) \overset{R2}{=} \log p - \log q$; $\log(pq/r) \overset{R1}{=} \log p + \log \frac{q}{r} \overset{R2}{=} \log p + \log q - \log r$; $\log(p/qr) \overset{R1\&2}{=} \log p - \log q - \log r$; $\log(p^3/q^2 r) \overset{R1\&2}{=} \log p^3 - \log q^2 - \log r \overset{R3}{=} 3\log p - 2\log q - \log r$.
$\log p + \log q \overset{R1}{=} \log pq$; $3\log p + \log q \overset{R3}{=} \log p^3 + \log q \overset{R1}{=} \log p^3 q$; $\log q - \log r \overset{R2}{=} \log \frac{q}{r}$; $3\log q + 7\log p \overset{R3}{=} \log q^3 + \log p^7 \overset{R1}{=} \log p^7 q^3$; $n\log p - \log q \overset{R3}{=} \log p^n - \log q \overset{R2}{=} \log \frac{p^n}{q}$.